Contents

Introduction vi
VELS correlation xi
VELS Level 5 Audit xiii
VELS Assignments xxi

1. Directed numbers
Home page: Into the negative zone 1
Prep Zone 2
Key words 2
1.1 Uses of directed numbers 3
Investigation: The shift game 5
1.2 Comparing directed numbers 6
1.3 Adding directed numbers 9
Problem solving: The slippery snail 12
1.4 Subtracting directed numbers 12
1.5 Simplifying addition and subtraction 15
VELS Design Task: Walking the plank 18
Problem solving: Pattern sums 20
1.6 Multiplication of directed numbers 20
1.7 Division of directed numbers 23
1.8 Combined operations 25
Laugh Zone 29
Maths in Action: The ultimate cool 30
Investigation: Positive/negative 32
Chapter Review 33
VELS Personal Learning Activity 33
Replay 37

2. Number techniques
Home page: Towards infinity 39
Prep Zone 40
Key words 40
2.1 Fractions with a calculator 41
Investigation: Estimating with fractions 44
2.2 Recurring decimals 45
Problem solving: Cyclic numbers 49
2.3 Mixing numbers 49
2.4 Squares and square roots of decimals and fractions 55
Investigation: How many seconds old are you? 57
2.5 Some other calculator functions 58
VELS Design Task: Finding √5 using fractions 63
Laugh Zone 64
2.6 Rates 65
Maths in Action: Being water wise 68

2.7 Indices 70
Investigation: The cube chain problem 73
Chapter Review 74
VELS Personal Learning Activity 74
Replay 76

Mixed revision one 78

3. Percentages
Home page: Get real! 83
Prep Zone 84
Key words 84
3.1 Estimating percentages 85
3.2 Converting percentages to fractions 88
3.3 Converting percentages to decimals 93
3.4 Converting fractions to percentages 95
3.5 Converting decimals to percentages 98
Laugh Zone 100
3.6 Expressing one amount as a percentage of another 101
3.7 Complementary percentages 106
3.8 Finding a percentage of an amount 108
3.9 Finding a percentage of an amount using decimals 111
Problem solving: Cordial contents 113
3.10 Business applications of percentages 114
Maths@Work: Accountant 117
3.11 Using a calculator to find percentages 118
Investigation: The plummeting price 122
VELS Design Task: Hitting pay-dirt 123
Chapter Review 124
VELS Personal Learning Activity 124
Replay 127

4. Algebra
Home page: Bubble algebra bursts onto the scene 129
Prep Zone 130
Key words 130
4.1 The meaning of pronumerals 131
4.2 Coefficients, constants and simplifying expressions 135
4.3 Substituting positive numbers 138
4.4 Substituting negative numbers 142
Investigation: The value of the Australian dollar 146
4.5 Formulae 147

4.6 Functions 148
4.7 Equations 169

4.8 Sequences 178

4.9 Graphs 181

4.10 Applications of algebra 186

4.11 Problem solving: Cordial contents 192

4.12 Preparing for VELS Level 6 193

Chapter Review 194
Appendix A: Answers 201
Appendix B: Index 209
8. Cartesian graphs
Home page: Chess moves all in the mind 339
Prep Zone 340
Key words 340
8.1 Interpreting points on a graph 341
8.2 Interpreting line graphs 346
8.3 The Cartesian plane 352
Maths in Action: The Cartesian coordinate system meets art! 358
8.4 Using tables to plot relationships 359
Investigation: Arm length and height 364
8.5 Plotting relationships using a rule 364
VELS Design Task: Graphical gymnastics 372
Investigation: Five in a line 373
8.6 Linear relationships 374
8.7 Finding the rule 378
Laugh Zone 384
8.8 Using linear relationships 385
Investigation: What a gem! 388
Chapter Review 389
VELS Personal Learning Activity 389
Replay 393

9. Geometry
Home page: Parallel Moon shadows meet! 395
Prep Zone 396
Key words 396
9.1 Exterior angle of a triangle 397
9.2 Angles on parallel lines 399
Laugh Zone 405
Computer investigation: Patterns and designs 406
9.3 Geometrical solids 407
9.4 Polyhedra 410
Investigation: The Möbius strip 412
9.5 Drawing and visualising 3D shapes 414
9.6 Nets and solids 417
9.7 Networks 424
Problem solving: The pipeline problem 427
Investigation: The game of sprouts 428
9.8 Traversable networks 428
9.9 Using networks 431
Maths@Work: Operations manager 433
VELS Design Task: Anna’s delivery route 434
Chapter Review 435
VELS Personal Learning Activity 435
Replay 440

10. Chance and data
Home page: Counting the catch 443
Prep Zone 444
Key words 444
10.1 Presenting data—histograms 445
10.2 Presenting data—computer-generated graphs 450
VELS Design Task: Petrol prices 455
10.3 Misuse of statistical graphs 456
10.4 Interpreting data 463
Maths in Action: Keeping it secret 471
10.5 Stem-and-leaf plots 473
Investigation: Do you measure up? 481
Laugh Zone 483
Computer investigation: Statistical measures 484
10.6 Probability (one-step experiments) 485
Problem Solving: Random walks 489
Graphics calculator investigation: Investigating the game of one-up 490
Chapter Review 492
VELS Personal Learning Activity 492
Replay 499

11. Structure
Home page: Achilles can’t beat the tortoise! 503
Prep Zone 504
Key words 504
11.1 Set notation 505
11.2 Truth functions 509
Computer investigation: More data 512
Maths in Action: Searching the ‘net’ 513
11.3 Identity elements and inverse operations 515
11.4 Correspondence and graphing 516
11.5 Inequalities 520
VELS Design Task: Checking for safety 522
Laugh Zone 523
Chapter Review 524
VELS Personal Learning Activity 524
Replay 527
Mixed revision four 528
Answers 536
Glossary and index 593

sample pages only
Welcome to *Heinemann Maths Zone VELS Edition*. This textbook and accompanying student CD and teacher support material are part of an exciting series that combines carefully crafted content written specifically for the Victorian Essential Learning Standards with a stunning, state-of-the-art, full colour design.

**Maths Zone VELS Edition textbook**

**VELS features**

- VELS Personal Learning Activities that specifically address the Physical, Personal and Social Learning Strand through student self-reflection
- VELS Design Tasks that specifically address the Interdisciplinary Learning Strand using rich tasks
- VELS Assignments that integrate all three Strands.

**Theory sections**

- Concise, uncluttered explanations
- Clear diagrams and graphs
- Danger Zones highlighting common errors students should watch out for
- Worked examples in Steps/Solutions format, which clearly separates instructions from working
- Highlight boxes providing a concise summary of major points

**Home pages**

- Motivating snippets of information related to the chapter
- Links to websites for further exploration

**Exercises**

- Exercises divided into Skills, Applications and Analysis questions
- Large numbers of carefully graded questions
- Explicit matches to Worked Examples

**Worked Example 1**

- Open-ended questions indicated by
- Multiple choice questions
- Bright, stimulating photos

Hi, I'm Minh. We're going to help along the way.

**sample pages only**
INTRODUCTION

Questions that help teachers ascertain if students have the prerequisite skills
References to Replay worksheets, which provide remediation
Key word list for mathematical literacy

Maths in Actions and Maths@Works

- Historical and real-life contexts for the mathematics being studied
- Weblinks to aid further research

Prep Zones

- Questions that help teachers ascertain if students have the prerequisite skills
- References to Replay worksheets, which provide remediation
- Key word list for mathematical literacy

Laugh Zones

- Cumulative mid-chapter revision
- Self-correcting code-based answers that reveal a cartoon caption

Investigations and Problem Solving tasks

- Group work icons where relevant
- Tasks related to the chapter content
- Specific problem solving strategies

Technology-based investigations

- Computer, graphics calculator and CAS investigations
- Key strokes for both TI and Casio models
- Computer Algebra Systems introduced
**Chapter reviews**

- Do-it-yourself chapter summaries
- Skills, Applications and Analysis questions
- References back to relevant textbook sections
- Cumulative revision Replays

**Mixed revisions**

- Multi-chapter cumulative revision
- Mixed Skills, Applications and Analysis questions
- References to relevant textbook sections
- Maths-competition style challenges

**Other Maths Zone features**

- Key terms bolded in text
- Answers with page references
- Glossary and index

---

**Heinemann eMaths Zone VELS Edition student CD**

*Heinemann eMaths Zone VELS Edition* is an interactive student CD that has been designed to complement the equivalent textbook. It contains the textbook in full, plus over 1000 dynamic links.

---

**eTutorials**

- Interactive multi-media presentations offering narrated alternative explanations and remedial support

---

**eQuestions**

- eQuestions providing extra on-screen practice
• Animations offering dynamic hints to specific questions

Animations

• eTesters that generate random race-against-the-clock mental mathematics tests

eTesters

• Interactives that are randomly generated concept developers

Interactives

• Pop-up hints to provide help with specific questions

Hints

Other Heinemann eMaths Zone VELS Edition features

• Starter and Restarter problems providing an overarching context for chapters

• Editable worksheets
• Replay worksheets designed for remediation
• Consolidation worksheets providing students with extra practice

• Homework sheets specifically tailored to the textbook
• Chapter Assignments for test revision

• Links to relevant websites
• Bonus software
Heinemann Maths Zone VELS Edition Homework Program

Heinemann Maths Zone VELS Edition Homework Program provides a series of tear-out homework sheets designed to complement the matching textbook.

- Full year’s homework program
- Cumulative revision sections
- Problem solving
- Questions tied to textbook Exercises.


Heinemann Maths Zone VELS Edition FlexiTest

FlexiTest provides a bank of over 1000 fully editable questions tied to textbook sections, and an easy-to-use program that allows you to create tests or revision sheets and matching worked solutions at the click of a mouse. Skills, Applications and Analysis questions are included.

Heinemann Maths Zone VELS Edition Website

The website www.hi.com.au/mathszonevels provides teachers with a complete package of VELS assessment and support, including:
- VELS audits in editable form
- VELS Assessment Tasks and detailed assessment notes
- Solutions to Starters and Restarters, homework sheets, chapter assignments and Consolidation worksheets
- Teacher’s notes to VELS Design Tasks, VELS Assignments, Investigations, Problem Solving tasks, Laugh Zones, Maths in Actions, Maths@Works, DIY summaries and Challenge Maths
**Heinemann Maths Zone 7 and 8 VELS Edition VELS Level 5 Audit**

The following Audit shows in detail where each of the Level 5 Victorian Essential Learning Standards for the Mathematics Discipline is covered in *Heinemann Maths Zone 7 VELS Edition* and *Heinemann Maths Zone 8 VELS Edition*. For other VELS-related material see www.hi.com.au/mathszonevels.

<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 7 and 8 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number Level 5</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Students identify complete factor sets for natural numbers and express these natural numbers as products of powers of primes. | 7 Ch 2 Ex 2.3, 2.4, 2.5  
7 Ch 2 Investigation: The sieve of Eratosthenes |
| They write equivalent fractions for a fraction given in simplest form (for example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \ldots$). | 7 Ch 3 Ex 3.1  
7 Ch 3 Investigation: Fraction wall |
| They know the decimal equivalents for the unit fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}$ and find equivalent representations of fractions as decimals, ratios and percentages (for example, a subset: set ratio of $4:9$ can be expressed equivalently as $\frac{4}{9} = 0.4444\ldots$). They write the reciprocal of any fraction and calculate the decimal equivalent to a given degree of accuracy. | 7 Ch 4 Ex 4.4  
7 Ch 4 Investigation: Fractions to decimals  
8 Ch 2 Ex 2.1, 2.3  
8 Ch 3 Ex 3.2, 3.3, 3.4 |
<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 7 and 8 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students use knowledge of perfect squares when calculating and estimating squares and square roots of numbers (for example, $20^2 = 400$ and $30^2 = 900$, so $\sqrt{700}$ is between 20 and 30). They evaluate natural numbers and simple fractions given in base-exponent form (for example, $5^2 = 25$ and $2\times5 = \frac{10}{2}$). They know simple powers of 2, 3, and 5 (for example, $2^5 = 32$, $3^4 = 81$, $5^3 = 125$). They calculate squares and square roots of rational numbers that are perfect squares (for example, $\sqrt{0.81} = 0.9$ and $\sqrt{\frac{16}{9}} = \frac{4}{3}$). They calculate cubes and cube roots of perfect cubes (for example, $\sqrt[3]{64} = 4$). Using technology they find square and cube roots of rational numbers to a specified degree of accuracy (for example, $\sqrt[3]{250} \approx 5.848$ to three decimal places).</td>
<td>7 Ch 2 Ex 2.6, 8 Ch 2 Ex 2.3, 2.4, 2.5</td>
</tr>
<tr>
<td>Students express natural numbers base 10 in binary form (for example, $42_{10} = 101010_2$), and add and multiply natural numbers in binary form (for example, $1011_2 + 1100_2$ and $1011_2 \times 111_2 = 111111_2$).</td>
<td>7 Ch 2 Ex 2.7</td>
</tr>
<tr>
<td>Students understand ratio as both set : set comparison (for example, number of girls : number of boys) and subset : set comparison (for example, number of girls : number of students), and find integer proportions of these, including percentages (for example, the ratio number of girls : the number of boys is $2 : 3 : 4 \times 6 = 40% : 60%$).</td>
<td>8 Ch 5 Ex 5.1, 5.2, 5.3, 5.4, 8 Ch 5 Investigation: Getting the ratio right</td>
</tr>
<tr>
<td>Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding (for example, $925 + 34 - 900 + 30 = 30$).</td>
<td>7 Ch 1 Ex 1.5, 7 Ch 1 Investigation: Close enough is good enough, 7 Ch 1 Ex 1.5, 1.7, 8 Ch 2 Investigation: Estimating with fractions</td>
</tr>
<tr>
<td>VELS Mathematics Standards</td>
<td>Heinemann Maths Zone 7 and 8 VELS Edition references</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------------------------------------------------</td>
</tr>
</tbody>
</table>
| Students use efficient mental and/or written methods for arithmetic computation involving rational numbers, including division of integers by two-digit divisors. They use approximations to π in related measurement calculations (for example, π × 5² = 25π = 78.54 correct to two decimal places). | 7 Ch 1 Ex 1.4, 1.5, 1.6, 1.7  
7 Ch 3  
7 Ch 4  
8 Ch 1  
8 Ch 6 Ex 6.5 |
| They use technology for arithmetic computations involving several operations on rational numbers of any size. | 8 Ch 1 Ex 1.8  
8 Ch 2 Ex 2.1, 2.5  
8 Ch 3 Ex 3.11 |
| **Space** | |
| At Level 5, students construct two-dimensional and simple three-dimensional shapes according to specifications of length, angle and adjacency. They use the properties of parallel lines and transversals of these lines to calculate angles that are supplementary, corresponding, allied (co-interior) and alternate. They describe and apply the angle properties of regular and irregular polygons, in particular, triangles and quadrilaterals. They use two-dimensional nets to construct a simple three-dimensional object such as a prism or a Platonic solid. They recognise congruence of shapes and solids. They relate similarity to enlargement from a common fixed point. They use single-point perspective to make a two-dimensional representation of a simple three-dimensional object. They make tessellations from simple shapes. | 7 Ch 7 Ex 7.5, 7.7  
7 Ch 9 Ex 9.2, 9.4, 9.7  
7 Ch 9 Investigation: Angle sum in a triangle  
7 Ch 9 Investigation: Angle sum in a polygon  
7 Ch 9 Investigation: Polyiamonds  
7 Ch 9 Maths in Action: Putting painting in perspective  
8 Ch 9 Ex 9.1, 9.2, 9.3, 9.4, 9.5, 9.6 |
| Students use coordinates to identify position in the plane. They use lines, grids, contours, isobars, scales and bearings to specify location and direction on plans and maps. They use network diagrams to specify relationships. They consider the connectedness of a network, such as the ability to travel through a set of roads between towns. | 7 Ch 6 Maths@Work: Taxi driver  
7 Ch 9 Ex 6.7, 6.8  
8 Ch 8 Ex 8.1, 8.2, 8.3  
8 VELS Assignment 4: Maps and more  
8 Ch 9 Ex 9.7, 9.8, 9.9  
8 Ch 9 Investigation: The game of sprouts  
8 Ch 9 Maths@Work: Operations manager  
8 Ch 9 VELS Design Task: Anna’s delivery route |
<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 7 and 8 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurement, chance and data</strong></td>
<td></td>
</tr>
<tr>
<td>Students measure length, perimeter, area, surface area, mass, volume, capacity, angle, time and temperature using suitable units for these measurements in context. They interpret and use measurement formulae for the area and perimeter of circles, triangles and parallelograms and simple composite shapes. They calculate the surface area and volume of prisms and cylinders.</td>
<td>7 VELS Assignment 2: Measuring up 7 Ch 5 8 VELS Assignment 4: Maps and more 8 VELS Assignment 2: Tangrams with a difference 8 Ch 6 Investigation: Measuring circumference 8 Ch 6 Investigation: Pick’s theorem 8 Ch 6 VELS Design Task: Chocolate trays 8 Ch 6 CAS investigation: Using measurement formulae</td>
</tr>
<tr>
<td>Students estimate the accuracy of measurements and give suitable lower and upper bounds for measurement values. They calculate absolute percentage error of estimated values.</td>
<td>7 VELS Assignment 2: Measuring up</td>
</tr>
<tr>
<td>Students use appropriate technology to generate random numbers in the conduct of simple simulations.</td>
<td>8 Ch 10 Graphics calculator investigation: Investigating the game of one-up</td>
</tr>
<tr>
<td>Students identify empirical probability as long-run relative frequency. They calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes. They use tree diagrams to investigate the probability of outcomes in simple multiple event trials. Students identify empirical probability as long-run relative frequency.</td>
<td>7 Ch 10 Investigation: Experimental probability 7 Ch 10 Ex 10.8 8 Ch 10 Ex 10.6 8 Ch 10 Investigation: Tree diagrams</td>
</tr>
<tr>
<td>Students organise, tabulate and display discrete and continuous data (grouped and ungrouped) using technology for larger data sets. They represent univariate data in appropriate graphical forms including dot plots, stem-and-leaf plots, column graphs, bar charts and histograms. They calculate summary statistics for measures of centre (mean, median, mode) and spread (range, and mean absolute difference), and make simple inferences based on this data.</td>
<td>7 Ch 10 Ex 10.2, 10.3, 10.4, 10.5, 10.6 7 Ch 10 Investigation: Using averages 7 Ch 10 Computer investigation: Collecting statistics 8 Ch 10 Investigation: Do you measure up? 8 Ch 10 Ex 10.1, 10.2, 10.3, 10.4, 10.5 8 Ch 10 VELS Design Task: Petrol prices 8 Ch 10 Computer investigation: Statistical measures</td>
</tr>
</tbody>
</table>
VELS Mathematics Standards | Heinemann Maths Zone 7 and 8
---|---
**Structure**

At Level 5, students identify collections of numbers as subsets of natural numbers, integers, rational numbers and real numbers. They use Venn diagrams and tree diagrams to show the relationships of intersection, union, inclusion (subset) and complement between the sets. They list the elements of the set of all subsets (power set) of a given finite set and comprehend the partial-order relationship between these subsets with respect to inclusion (for example, given the set \{a, b, c\} the corresponding power set is \{Ø, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}).

They test the validity of statements formed by the use of the connectives and, or, not, and the quantifiers none, some and all, (for example, ‘some natural numbers can be expressed as the sum of two squares’). They apply these to the specification of sets defined in terms of one or two attributes, and to searches in databases.

Students apply the commutative, associative, and distributive properties in mental and written computation (for example, \(24 \times 60\) can be calculated as \(20 \times 60 + 4 \times 60\) or as \(12 \times 12 \times 10\)). They use exponent laws for multiplication and division of power terms (for example \(2^3 \times 2^5 = 2^8\), \(2^0 = 1\), \(2^3 = 2^5 = 2^2\), \((5^3)^1 = 5^3\) and \((3 \times 4)^2 = 3^2 \times 4^2\).

Students generalise from perfect square and difference of two square number patterns (for example, \(25^2 = (20 + 5)^2 = 400 + 2 \times (100) + 25 = 625\). And \(35 \times 25 = (30 + 5) (30 - 5) = 900 - 25 = 875\))

Students recognise and apply simple geometric transformations of the plane such as translation, reflection, rotation and dilation and combinations of the above, including their inverses.

---

8 Ch 1 Ex 1.1
8 Ch 11 Ex 11.1
8 Ch 11 Ex 11.2
8 Ch 11 Maths in Action: Searching the ‘net’
8 Ch 11 Computer investigation: More data
8 Ch 2 Ex 2.7
8 Ch 4 Ex 4.9, 4.10
8 Ch 2 Investigation: Number pattern games
7 Ch 9 Ex 9.6
7 Ch 9 Computer investigation: Using Microworlds or LOGO
They identify the identity element and inverse of rational numbers for the operations of addition and multiplication (for example, $\frac{1}{2} + \frac{1}{2} = 0$ and $\frac{3}{4} \times \frac{4}{3} = 1$).

Students use inverses to rearrange simple mensuration formulae, and to find equivalent algebraic expressions (for example, if $P = 2L + 2W$, then $W = \frac{P}{2} - L$. If $A = \pi r^2$ then $r = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$).

They solve simple equations (for example, $5x + 7 = 23$, $4x - 1.6 = 8.5$, and $4x^2 - 3 = 13$) using tables, graphs and inverse operations. They recognise and use inequality symbols. They solve simple inequalities such as $y \leq 2x + 4$ and decide whether inequalities such as $x^2 > 2y$ are satisfied or not for specific values of $x$ and $y$.

Students identify a function as a one-to-one correspondence or as a many-to-one correspondence between two sets. They represent a function by a table of values, a graph, and by a rule. They describe and specify the independent variable of a function and its domain, and the dependent variable and its range. They construct tables of values and graphs for linear functions. They use linear and other functions such as $f(x) = 2x - 4$, $xy = 24$, $y = 2^x$ and $y = x^2 - 3$ to model various situations.

<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 7 and 8 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td>They identify the identity element and inverse of rational numbers for the operations of</td>
<td>8 Ch 11 Ex 11.3</td>
</tr>
<tr>
<td>addition and multiplication (for example, $\frac{1}{2} + \frac{1}{2} = 0$ and  (\frac{3}{4} \times \frac{4}{3} = 1))</td>
<td></td>
</tr>
<tr>
<td>Students use inverses to rearrange simple mensuration formulae, and to find equivalent</td>
<td>8 Ch 4 VELS Design Task: Medicinal doses</td>
</tr>
<tr>
<td>algebraic expressions (for example, if $P = 2L + 2W$, then $W = \frac{P}{2} - L$. If $A = \pi r^2$ then $r = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$)</td>
<td>8 Ch 4 Maths in Action: The algebra of eclipses</td>
</tr>
<tr>
<td>They solve simple equations (for example, $5x + 7 = 23$, $4x - 1.6 = 8.5$, and $4x^2 - 3 = 13$) using tables, graphs and inverse operations.</td>
<td>8 Ch 4 Problem solving: Garden sleepers</td>
</tr>
<tr>
<td>They recognise and use inequality symbols. They solve simple inequalities such as $y \leq 2x + 4$ and decide whether inequalities such as $x^2 &gt; 2y$ are satisfied or not for specific values of $x$ and $y$.</td>
<td>8 Ch 4 Ex 4.9</td>
</tr>
<tr>
<td>Students identify a function as a one-to-one correspondence or as a many-to-one</td>
<td>8 Ch 11 Ex 11.3</td>
</tr>
<tr>
<td>correspondence between two sets. They represent a function by a table of values, a graph,</td>
<td></td>
</tr>
<tr>
<td>and by a rule. They describe and specify the independent variable of a function and its</td>
<td></td>
</tr>
<tr>
<td>domain, and the dependent variable and its range. They construct tables of values and</td>
<td></td>
</tr>
<tr>
<td>graphs for linear functions. They use linear and other functions such as $f(x) = 2x - 4$,</td>
<td></td>
</tr>
<tr>
<td>$xy = 24$, $y = 2^x$ and $y = x^2 - 3$ to model various situations.</td>
<td></td>
</tr>
<tr>
<td>Working mathematically</td>
<td></td>
</tr>
<tr>
<td>--------------------------------------------------------------------------------------</td>
<td></td>
</tr>
</tbody>
</table>
| Students formulate conjectures and follow simple mathematical deductions (for example, if the side length of a cube is doubled, then the surface area increases by a factor of four, and the volume increases by a factor of eight). | 7 Ch 1 Investigation: Palindromes  
7 Ch 2 Computer investigation: Fibonacci and other number patterns  
7 Ch 3 Investigation: Ideal fractions  
7 Ch 5 Investigation: How many squares on a chessboard?  
7 Ch 6 Investigation: Cups and counters  
8 Ch 2 Investigation: The cube chain problem  
8 Ch 6 Investigation: Measuring circumference  
8 Ch 8 Investigation: Arm length and height  
8 Ch 9 Investigation: The game of sprouts |
| Students use variables in general mathematical statements. They substitute numbers for variables (for example, in equations, inequalities, identities and formulae). | 7 Ch 6, Ch 8  
8 Ch 7, Ch 8  
8 Ch 11 Ex 11.5 |
| Students explain geometric propositions (for example, by varying the location of key points and/or lines in a construction). | 7 Ch 9 Investigation: Angle sum in a triangle  
7 Ch 9 Investigation: Angle sum in a polygon  
7 Ch 9 Computer investigation: Using Microworlds or LOGO  
8 Ch 9 Computer investigation: Patterns and designs |
| Students develop simple mathematical models for real situations (for example, using constant rates of change for linear models). They develop generalisations by abstracting the features from situations and expressing these in words and symbols. They predict using interpolation (working with what is already known) and extrapolation (working beyond what is already known). They analyse the reasonableness of points of view, procedures and results, according to given criteria, and identify limitations and/or constraints in context. | 7 VELS Design tasks  
8 Ch 5 Investigation: Russian multiplication  
8 Ch 7 Ex 7.8, 7.9  
8 Ch 7 Investigation: Peter’s gardening service  
8 Ch 8 Ex 8.8  
8 Ch 8 Investigation: What a gem! |
Students use technology such as graphic calculators, spreadsheets, dynamic geometry software and computer algebra systems for a range of mathematical purposes including numerical computation, graphing, investigation of patterns and relations for algebraic expressions, and the production of geometric drawings.

<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 7 and 8 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Ch 2 Computer investigation: Fibonacci and other number patterns</td>
<td>7 Ch 6 CAS investigation: Exploring number patterns</td>
</tr>
<tr>
<td>7 Ch 6 Graphics calculator investigation: Algebraic mind reading</td>
<td>7 Ch 6 Graphics calculator investigation: Hypatia and Diophantine equations</td>
</tr>
<tr>
<td>7 Ch 8 Graphics calculator investigation: Hypatia and Diophantine equations</td>
<td>7 Ch 8 Computer investigation: Solving equations using substitution</td>
</tr>
<tr>
<td>7 Ch 9 Computer investigation: Using Microworlds or LOGO</td>
<td>7 Ch 9 Computer investigation: Collecting statistics</td>
</tr>
<tr>
<td>8 Ch 6 CAS investigation: Using measurement formulae</td>
<td>8 Ch 9 Computer investigation: Patterns and designs</td>
</tr>
<tr>
<td>8 Ch 10 Computer investigation: Statistical measures</td>
<td>8 Ch 10 Graphics calculator investigation: Investigating the game of one-up</td>
</tr>
<tr>
<td>8 Ch 11 Computer investigation: More data</td>
<td></td>
</tr>
</tbody>
</table>
VELS Assignments

Assignment 1: Physical numbers

Chapter references: 1 & 2

You will need: cards (one per student) and group leaders

Students form into small groups, each with a leader.

1. Each student draws two lines in the top left-hand corner of the card to form a box as shown below:

2. The leader writes the word LEADER at the bottom of their card, then puts a number in the corner box. It can be negative, positive, a fraction or decimal (or chosen according to restrictions specified by the teacher).

3. Then, in view of the group, the leader writes an operation to be performed on this number.

4. When the group agrees on an answer, that number is written in the corner box of the next student’s card.

5. This process continues but with the next student individually putting a question and writing the answer in the next person’s corner box.

6. The last person, however, needs to work out a question whose answer is the number in the leader’s box. He/she may decide to ask for the group’s help.

7. The group then checks that the questions and answers are correct and that none of the numbers in the corner boxes are the same.

8. The cards are shuffled and each group swaps their set with another group.

9. Then each group will need to work out the correct order of the cards.

Note: if you want to make this difficult for the other groups, put hard questions on your card!

Assignment 2: Tangrams with a difference

Chapter references: 3, 5 & 6

You will need: pieces of firm paper or cardboard
1. Mark a square that is 8 cm × 8 cm on the piece of paper and, with or without the use of a grid, draw the lines as shown in the diagram. This is called a tangram.

2. Find the area of each of the shapes using the grid to assist you.

3. Show that the values you found in 2 can also be calculated by using the formulae you have learnt.

4. Now draw up a tangram in a 4 cm × 4 cm square. Find the corresponding areas. What do you notice?

5. Individually or with a partner brainstorm about the relationships between the shapes, e.g. the small triangle’s area is 50% of the small square in the same tangram, the ratio of the area of the two big triangles to the whole square is 1 : 2. Find as many relationships as you can, including between the shapes of the two tangrams.

6. Cut along the lines of both tangrams and, with the pieces, form different shapes. Before cutting you may like to make a note of where the pieces came from.

7. Draw around the shapes you have made on a different piece of paper and swap with a partner. Now try to find the area of the shapes drawn. This can be done by using the original shapes or by calculating the area.
Assignment 3: People graphs

Chapter reference: 8

You will need: an open space with a hard surface and chalk

1. Draw a set of axes on the surface and mark points at equal distances along it, e.g. a small stride apart. If you can’t draw on the ground, use two pieces of laid-out string.

2. Each person should be a particular $x$ value, depending on how many are participating, but there should be at least one person each for $x = -1, 0, 1, 2, 3, 4, 5$.

3. First, practice moving on the graph with a few coordinates, e.g. $(2, 4)$ and $(-1, 3)$; the person who is $x = 2$ moves to where $y = 4$ and the person who is $x = -1$ moves to where $y = 3$, etc.

4. You will now form a straight line along the pattern that $y$ is twice $x$.

5. The person for $x = 0$ will stand at the origin. $x = 1$ will have to find where $y$ is double this. It is best to do this one at a time.

6. Notice the pattern formed as you move across one at a time. What happens when you move one stride left to $x = -1$?

7. Try another pattern, this time starting when $x = 0, y = 1$.

8. How would you describe the pattern you have formed?

9. Can you identify who is the person representing the $x$-intercept, and who is representing the $y$-intercept?

10. When you are forming a pattern, what is the gradient? How can you form a negative gradient?

11. If you form some of the equations that you have looked at in class, would you get the same gradient and $x$- and $y$-intercepts as you did on paper?
Assignment 4: Maps and more

Chapter reference: 8

You will need: a street directory, atlas and weather map from the newspaper

In this assignment you will be investigating ways of describing places, positions, heights and climate.

1. Think of a map of Australia. If the area of Victoria has a value of 1, then assign a relative value to each of the other states and territories. For example, if you think a state is twice the size of Victoria, you would assign it a 2, if you thought it was half its size then you would assign it a $\frac{1}{2}$. Draw up a table with nine rows and six columns. Place the names of the states in the first column and your estimate in the second column with the heading ‘First estimate’.

2. Look at a map of Australia and redo the exercise from 1, placing your results in the third column with the heading ‘Second estimate’.

3. Using the map of Australia and/or other resources, find out the area of each state.

4. Divide the area of each of the other states by the area of Victoria. This will give you the actual value that you should have found in 1 and 2. Place these numbers in the fourth column with the heading ‘Actual’.

5. You are now going to calculate the absolute percentage error for your results. The formula is:

$$\text{absolute percentage error} = \frac{\text{estimated value} - \text{actual value}}{\text{actual value}} \times 100$$

Fill in the last two columns of your table with the absolute percentage errors for each of your estimates.

6. What do you notice? Which state size was it hardest to predict?

7. The height of land is sometimes shown on a topographical map with lines showing joining locations of the same height. Using a topographical map, find all the areas in Victoria that are 1000 m above sea level. If you can’t find a topographical map of Victoria then use one of a different area.

8. Another way to read a map is to use its grid to identify where a place is located. Using your map of Australia or a map of Victoria, write down the names of at least three places and their grid references. By convention the horizontal gridline is read first, followed by the vertical (usually a letter followed by a number).

9. Look at the weather map in the newspaper and identify its key features. The isobars are lines that link places with equal air pressure. If the isobars are close together then the wind is stronger. Identify places on the map where you think the wind will be stronger.
Assignment 5: What the labels say

Chapter references: 2, 3 & 10

You will need: hamburger wrappers or manufactured food products labels that contain nutritional information

1. Draw up a table with the headings: Product; Energy (kJ); Protein (g); Fat Total (g); Carbohydrates Total (g); Sugars (g) and Dietary Fibre (g).

2. From each of the food labels, find the values corresponding to these headings. If the serving sizes differ, you will need to express each of these values as percentages. (You can put this table into a spreadsheet.)

3. Draw a graph of each of the individual products.

4. Produce another graph that compares all the products.

5. Based on your graphs, which product do you think is the most healthy? Give reasons for your answer.

6. Determine the percentage of fat in each of the products; how does it compare to any advertising about the product or to what you had thought about the fat content of the product?

7. Find out what is the recommended daily intake of energy for you; what percentage of this is supplied by a serve of each product? How many serves would you need to meet your daily requirement? If you had this many serves what do you think you would have too much of?
A story from Ancient Greece tells of a proposed race between Achilles and a tortoise: Since Achilles can run 10 times as fast as the tortoise, the tortoise was to be given a 10 m start. Everyone knew that Achilles would easily catch up with the tortoise, but the wise philosopher, Zeno, put up an argument that made people unsure. Zeno argued that, if the tortoise had a 10 m lead, by the time Achilles had run those 10 m, the tortoise would have moved forward another 1 m, then as Achilles covered that 1 m, the tortoise would move forward further still (by 10 cm), and so on, so that Achilles would never be able to catch up with the tortoise! There is something wrong with the logic of this argument. Can you show what it is?
Prepare for this chapter by attempting the following questions. If you have difficulty with a particular question, click on its Replay Worksheet icon on your eMaths Zone CD or ask your teacher for the Replay Worksheet.

1 On a Cartesian plane plot the following coordinates: (1, 2), (-3, 4), (2, 2), (-3, -1)

2 For the numbers 1–20 inclusive find:
   (a) all the even numbers
   (b) all the prime numbers
   (c) all the numbers divisible by 6

3 By setting up a table of values from -3 to 3, graph each of the following on a separate graph.
   (a) \( y = x + 1 \)  
   (b) \( y = 2x \)  
   (c) \( y = -x + 2 \)

4 State whether each of the following statements is always true, sometimes true or never true.
   (a) Geelong won when they played Richmond last time in the AFL.
   (b) The sun rose this morning.
   (c) The weather today is cold.
   (d) \( 2 + 2 = 4 \)
   (e) In Australia it is not against the law to steal.

5 Find the value of \( x \) in each of the following:
   (a) \( x + 2 = 5 \)  
   (b) \( 2x = 16 \)  
   (c) \( x - 3 = 7 \)

**KEY WORDS**

- complement
- domain
- function
- identity element
- inequality
- integers
- intersection
- natural numbers
- null set
- power set
- range
- rational numbers
- real numbers
- set
- subset
- tree diagram
- union
- universal set
- Venn diagram
A set is a collection of numbers or objects. In mathematics, we use symbols to make things shorter, and thus to show features that relate sets together. A subset is part of a set. For example, if a set is all the students in the class, a subset would be the female students in the class. If there are no female students, then this subset would have nothing in it. We call this the null set, shown as ∅. A power set is the set that contains all the subsets. So if we have the set {1, 2, 3}, possible subsets are {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}. Each of these subsets is contained within the power set.

A Venn diagram is a picture or diagram that can be used to represent sets and their relationship to each other. In a Venn diagram the elements within the rectangular border represent the universal set, often shown as ε. The universal set is the set of all elements. Other subsets are placed inside circles within the border.

This Venn diagram shows the set ε = {2, 4, 6, 8, 10, 12, 14, 16, 18} and the subset A = {4, 6, 8, 12, 16}. To show that A is a subset of ε we write A ⊂ ε. Or, we can say that A is included in ε.

The complement of set A is the set of elements inside the universal set that is not in A. The complement of A can be shown as A′ or A. The set A′ contains the elements {2, 10, 14, 18}, i.e. the elements of the universal set not in A.

**worked example 1**

(a) Draw a Venn diagram to represent the following subsets of ε = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}: A = {1, 3, 5, 7}, B = {2, 4, 6, 8, 10}

(b) Find A′

**Steps**

(a) 1. Draw the inside circles first. There should be two circles, one for set A and one for set B. Write the elements inside the circles.
2. Place a rectangle around the circles and put ε in the corner. Write in the rectangle any elements that haven't been included in the circles.

(b) A′ is the set of all elements in ε that isn’t in A. Use set notation.

**Solutions**

(a) A = {1, 3, 5, 7} B = {2, 4, 6, 8, 10}

(b) A′ = {2, 4, 6, 8, 9, 10}
The union of two or more sets is the combining of the sets and contains every element of the original sets. The union of $A$ and $B$ is written as $A \cup B$. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ then $A \cup B = \{1, 2, 3, 4, 5\}$.

You may have noticed that $A$ and $B$ had an element in common, ‘3’. This is not written twice in the union because it is just an element that is common to both, or in the intersection of both sets. The intersection of sets $A$ and $B$ is written as $A \cap B$ and just comprises those elements that are common to both sets.

These concepts can be clearly shown using a Venn diagram.

**worked example 2**

For the sets $A = \{\text{factors of 15}\}$, $B = \{\text{factors of 24}\}$, where the universal set is the set of positive integers up to and including 24:

(a) Draw a Venn diagram showing all the sets.
(b) Find $A \cap B$
(c) Find $A \cup B$
(d) $A'$

**Steps**

(a) 1. List all the elements of the universal set.

2. List the elements of each of the other sets.

3. Determine the elements in common, if any.

4. Draw the Venn diagram remembering to include all elements in the universal set.

(b) Find $A \cap B$ by looking at the elements that are in the overlapping part of the Venn diagram, i.e. that are common to both sets.

(c) Find $A \cup B$ by including all the numbers that are in the two circles.

(d) $A'$ is all the elements not in the subset $A$.

**Solutions**

Positive integers up to and including 24

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$

$A = \{1, 3, 5, 15\}$

$B = \{1, 2, 3, 4, 6, 8, 12, 24\}$

\{1, 3\} are common to $A$ and $B$

(b) $A \cap B = \{1, 3\}$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 12, 15, 24\}$

(d) $A' = \{2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$
In the same way as a Venn diagram can be used to represent relationships between sets, so can a **tree diagram**. A tree diagram starts with the universal set and then branches to the other subsets. Let us look at the real number system within which there are a number of categories. All numbers that you are aware of are called **real numbers**. Numbers that are whole numbers are called **integers** i.e. \( \ldots, -3, -2, -1, 1, 2, 3, \ldots \). **Natural numbers** are numbers that are positive integers i.e. 1, 2, 3, 4, \ldots. Therefore natural numbers are a subset of integers. A **rational number** is a number that can be written as a fraction, i.e. in the form \( \frac{a}{b} \) where \( b \neq 0 \), so the set includes all the fractions that you know, even those that are recurring decimals. However, numbers such as \( \sqrt{2} \) and \( \pi \) cannot be written as fractions and so are irrational. Therefore rational numbers are a subset of real numbers and integers are a subset of rational numbers. These relationships can be shown on a tree diagram:

![Tree Diagram](attachment:image.png)

or as a Venn diagram.

![Venn Diagram](attachment:image.png)

### exercise 11.1  Set notation

**Skills**

1. **(a)** Draw a Venn diagram to represent the following subsets of \( \varepsilon = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} \):
   - \( A = \{2, 4, 6, 8\} \), \( B = \{8, 10, 12, 14, 16, 18\} \)
   - Find \( A' \).

2. **(a)** For the sets \( M = \{\text{factors of 20}\} \), \( N = \{\text{factors of 12}\} \) and the whole set of positive integers up to and including 24:
   - **(a)** draw a Venn diagram showing the sets
   - **(b)** find \( M \cap N \)
   - **(c)** find \( M \cup N \)
   - **(d)** find \( M' \).

\( A' \) means the complement of \( A \).
For the set \( A = \{ -2, 4, 6, 7, \sqrt{3}, \sqrt{9} \} \), determine which elements are:

(a) rational numbers
(b) integers
(c) natural numbers

4 Draw a Venn diagram to illustrate each of the following:

(a) \( \varepsilon = \{ \text{natural numbers less than 10} \}, A = \{ 1, 3, 5, 7, 9 \}, B = \{ 2, 4, 6, 8 \} \)
(b) \( \varepsilon = \{ 2, 4, 5, 6, 8, 10 \}, A = \{ 2, 6, 10 \}, B = \{ 5 \} \)
(c) \( \varepsilon = \{ \text{integers between and not including 0 and 7} \}, A = \{ 2, 6 \}, B = \{ 1, 2, 3, 4, 5 \} \)

5 Find the following from each of the Venn diagrams from 4.

(i) \( A \cap B \)  
(ii) \( A \cup B \)  
(iii) \( A' \)  
(iv) \( B' \)  
(v) \( (A \cap B)' \)

6 Determine all the possible subsets of the set \( \{ a, b, c, d \} \)

7 The set \( \{ 1, 2, 3 \} \) is the result of an intersection of two sets. Make up possibilities for the original two sets. Then find the union of the two sets and draw a Venn diagram to show the relationships. Using the notation that you have learnt, find as many relationships as you can between the sets.

8 For the universal set \( \varepsilon = \{ -3, -2, -1, 0, 1, 2, 3 \} \)

(a) The subset containing natural numbers is:
\[ A = \{ -3, -2, -1, 0, 1, 2, 3 \}, \quad B = \{ -3, -2, -1, 1, 2, 3 \}, \quad C = \{ 0, 1, 2, 3 \}, \quad D = \{ 1, 2, 3 \}, \quad E = \{ -1, 0, 1, 2, 3 \} \]
(b) If \( A = \{ 1, 2, 3 \} \) and \( B = \{ -1, 0, 1, 2 \} \), then \( A \cap B \) is:
\[ A = \{ -1, 0, 1, 2, 3 \}, \quad B = \{ 1, 2, 3 \}, \quad C = \{ -1, 0, 1, 2 \}, \quad D = \{ 1, 2 \}, \quad E = \{ -1, 1, 2, 3 \} \]

**Applications**

9 Kara and Emma are trying to produce different patterns with four types of beads: \( A, B, C \), and \( D \).

(a) Determine all the possible ways they could group the beads, with a maximum of one bead of each type per group.

(b) How many different ways would be possible if each group had only two beads of different types in it?

10 (a) List the universal set that contains all the possible results of rolling a die.

(b) If set \( A \) is of the results that are even and set \( B \) is of results that are less than 3, draw a Venn diagram to show the situation.

(c) Draw a tree diagram to show all the possible results if a coin is tossed twice.
Analysis

11 In a class of 25 students, 15 students like both sport and music, 3 students do not like either music or sport and 6 students like only sport.

(a) Draw a Venn diagram to show this situation with the number of students in each position.
(b) Find the number of students who like either sport and/or music.
(c) How many students like only music?

11.2 Truth functions

Statements can be made about sets using words such as or and and. For example, the statement ‘contained within the set of even positive integers or the set of odd positive integers’ combines the set of even positive integers with the set of odd positive integers, or shows the union of the two sets. Another way to express this would be ‘contained within the set of positive integers’. The two statements are said to be equivalent because the resulting set is the same. The statement ‘in one set and in another set’ implies that the resulting set is contained within the intersection of the two. Other relevant words are none, some and all. For example: none of the elements of the set of even numbers is contained within the set of odd numbers; some even numbers are positive; all even numbers are divisible by 2.

worked example 3

For the sets $A = \{\text{the letters of the word ALPHABET}\}, B = \{A, P, T\}, C = \{L, H, B, E\}$

(a) find the set containing all the elements that are in $B$ and $C$
(b) find the set containing all the elements that are in $A$ or $B$
(c) State an equivalent set to the answer to (b).

Steps

(a) $B$ and $C$ means elements that are part of both sets.
(b) $A$ or $B$ means the elements that are in either of the two sets combined or the union of the two sets.
(c) Two sets are equivalent if they have the same elements.

Solutions

(a) there are none $\therefore \emptyset$
(b) $\{A, P, T, L, H, B, E\}$
(c) $A$ is equivalent to ‘$A$ or $B$’ since it contains the same elements.

In the worked example above, since $A$ or $B$ resulted in the set $A$, it can be said that $B$ is a subset of $A$ as all its elements are contained within $A$. If only some of $B$ was contained within $A$ then it wouldn’t be a subset.
The example above also illustrates what is known as the truth value of a statement, but this is not the only way to show the truth of a statement. A truth function uses a ‘1’ for a statement that is true and a ‘0’ for a false statement. The use of truth functions allows deductions to be made.

worked example 4

Anita, Greg and Fan all have jobs. One of them is a plumber, one a teacher and one a computer programmer. Anita dislikes computers. Greg has never had anything to do with children. Greg did an apprenticeship to be trained for his job. Using a truth table determine the job each person does.

**Steps**

1. Set up a truth table.

**Solution**

<table>
<thead>
<tr>
<th></th>
<th>plumber</th>
<th>teacher</th>
<th>computer programmer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anita</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Greg</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Fan</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Anita cannot be the computer programmer because she dislikes computers.

Greg cannot be the teacher as he has never had anything to do with children.

An apprenticeship is needed for a plumber but not for the other two jobs so Greg must be a plumber.

<table>
<thead>
<tr>
<th></th>
<th>plumber</th>
<th>teacher</th>
<th>computer programmer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anita</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Greg</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fan</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
**exercise 11.2 Truth functions**

**Skills**

1. For the sets $A = \{\text{even numbers between 2 and 16 inclusive}\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{10, 12, 14, 16\}$
   
   (a) find the set consisting of all the elements that are in $B$ and $C$
   
   (b) find the set consisting of all the elements that are in $A$ or $C$
   
   (c) State an equivalent set to the answer to (b).

2. Jason, Ruby and Abby each play only one sport at school. One plays hockey, one netball and one basketball. Ruby does not play basketball. Abby plays a sport that uses a stick. Using a truth table determine which sport each person plays.

3. A set contains the names Jane, Fred, Long, Jye and Fee.
   
   (a) Make a statement about this set that uses the quantifier ‘some’.
   
   (b) Make a statement about this set that uses the quantifier ‘all’.
   
   (c) Make a statement about this set that uses the quantifier ‘none’.
   
   (d) Add an element to the set that changes the statement in (c) from ‘none’ to ‘some’.

4. Anita plays netball, Mark plays football and Geoff plays tennis. Which of the following statements may be incorrect?
   
   A. All three people play a sport.
   
   B. Only one of the people play a game with a racquet.
   
   C. Mark is the best at sport.
   
   D. Anita and Mark play team games.
   
   E. Anita and Geoff play games involving balls.

**Applications**

5. A game requires you to find out who has committed each of three murders with three different weapons. The three people involved are Chris, Jason and Matt. The three weapons are a gun, a knife and a fist. The three murder victims are Rod, Terry and Jules. Chris did not use the gun. Jules was killed with a knife. Jason used a fist on his victim. Rod was shot. Using truth functions determine who was killed by which weapon.

**Analysis**

6. You know that the following is true: Troy is a chef who lives in Melbourne, Maria is a teacher who lives in Newcastle and Geoff is a computer programmer who lives in Lorne. Write at least one set of statements that can be used to work out this information, using the least number of clues possible.
**More data**

In this exercise you are going to collect information about people within your class and put it into a database. You are then going to be able to search the database for selected information.

1. Everyone in the class needs to write down the following information on a piece of paper: first name, surname, date of birth, favourite food, favourite song, how many hours a day they spend on average on the Internet, and whether they are male or female.

2. To put the information into a database, you will first need to set it up. Open a new database file and set up the following fields with the type as stated.

3. Using the information from the pieces of paper, enter the data into your database. Make sure that the data is put in under the same generic name, e.g. pizza and Tropicana pizza should both be entered as pizza.

4. Check that you know how to do a report using the database program that you have and print out a simple report of the information about the class. Check that there are no spelling or typing errors in the information.

5. Go to query in your database program and set up a query that shows the first name, surname and all those students whose favourite food is pizza (or a different food). This will usually require something like = ‘pizza’ with the appropriate field chosen. Produce a report to show the result.

6. More than one field can be specified to narrow down the information obtained from a database. This is known as a Boolean search, which uses and, or, or not to obtain the required information. For example, to make a list of the boys in the class whose favourite food is pizza, the query will require that the field ‘Favourite food’ contains ‘pizza’ and the field ‘Male or female’ contains ‘male’. Complete this query and produce a report to show the result.

7. Use the Date of birth and the Time on Internet in at least one query to find out particular information and produce the appropriate reports.

8. Not all information needs to be entered into a database. Sometimes new fields can be produced by using the existing fields. For example, you may want a field that would qualify Internet usage as ‘too much’ if a person spends more than 8 hours on the Internet and as ‘OK’ if it’s 8 hours or less. Use the fields ‘First name’ and ‘Surname’, and create a new field called ‘Internet usage’ – this is usually done on most database programs by putting the name in the field name followed by a colon. The expression will then need to work out that if time spent on the Internet is greater than 8 the column should show ‘too much’ and if it is less than or equal to 8 it should show ‘OK’. For Access, the expression would be: Internet usage: if (‘Time on Internet’ > 8, ‘too much’, ‘ok’). This expression in words is: If the field ‘Time on Internet’ has a value that is greater than 8 then write ‘too much’, else write ‘ok’. This type of expression is called an if…then…else.

9. Make up at least two different reports using queries and the skills that you have learnt above.
There is now so much information on the Internet, with billions of sites, each with a huge number of pages, that it has become necessary to use a search engine. A search engine provides a means of finding information that relates to the topic you want to find out about. It commonly uses ‘and’ to search for things where there is more than one component. Search engines have to achieve their results as quickly as possible and so they have to use the best possible algorithm. An algorithm is a set process that is followed every time. The popularity of a search engine is dependent on its speed and accuracy. Not only does a search engine need to find the search item, but it needs to find it in a meaningful context. Then when it presents the search results, it needs to sort them in an order in which the most appropriate result appears first. The most obscure occurrence of the required words should be last.
The way this is achieved is through a process of searching for all occurrences of all the words that are required and then finding their intersection. In other words, the search engine finds where the words all occur in the same document. The ordering of sites is very significant for both the user and for any site that wants to be viewed. Different search engines will use different conditions but important considerations are: how often a site is updated, how many links it has to it and what sort of reputation a web site has. This is very important because approximately 95% of people will only look at the first page of search results. There are also ways of making a web site more likely for a search engine to find it and give it a higher ranking for more keywords.

Searching so many pages quickly means that conditions are incorporated in very complicated queries. In fact Google uses an algorithm called Page Rank that incorporates 500,000,000 variables and more than 2 billion terms.

Questions

1. Explain in your own words how you think a search engine will find something like 'labradoodle dog' that requires searching for two related words.

2. Write an expression that uses If… then… else to help decide which web site should be ranked higher than others.

3. Do a search on the Internet on 'maths and computers'—note the names of the top 5 web sites and any other information provided by the search engine.

4. Do a search on the Internet on 'computers and maths'—note the names of the top 5 web sites and compare with your results from 3.

5. Why do you think that search engines want to be the ones that people use?

Research

1. Investigate at least two different search engine sites and what they have to say about what they do and how they do it. You can even try to search for the same item, using the two search engines, and compare the results.

2. Investigate what ‘Google bombing’ is.
An identity element for a particular operation is an element that does not change the value of the number to which it is applied. Adding 0 to a number doesn’t change any number. You can always add 0 and the number will remain the same; this isn’t true for any other number when adding. However, with multiplication, if we multiply by 0 we change the value of the number to 0 (unless the original number was 0). For multiplication the identity element is 1 as you can always multiply by 1 and the value will not be changed.

The inverse of a particular operation is the one that produces the identity element for that operation. So the inverse of adding 4 would be subtracting 4 because adding 4 and then subtracting 4 gives 0, which is the identity element for addition. The inverse operation for multiplying by 4 would be dividing by 4 as this would give 1, the identity element for multiplication.

**worked example 5**

Find the inverse operation for each of the following and show that the identity element results.

(a) multiplying by 3  
(b) adding -3

**Steps**

(a) 1. The inverse of multiplication is division.
    2. The identity element for multiplication should result.

(b) 1. The inverse of addition is subtraction.
    2. The identity element for addition should result.

**Solutions**

(a) Divide by 3.  
    \( \times 3 + 3 = 1 \), the identity element for multiplication

(b) Subtract -3, which is the same as adding 3.  
    \( -3 + 3 = 0 \), the identity element for addition

**exercise 11.3 Identity elements and inverse operations**

**Skills**

For each of the following find the inverse operation and show that the identity element for that operation results.

(a) adding 4  
(b) multiplying by 3  
(c) multiplying by -2  
(d) adding -1  
(e) adding \( \frac{1}{2} \)  
(f) multiplying by 1.5  
(g) subtracting -2  
(h) dividing by 3  
(i) dividing by \( -\frac{3}{4} \)
2 Determine whether each of the following statements is true or false.
   (a) The inverse operation for subtracting \(-3\) is adding \(-3\).
   (b) The identity element for multiplication is 0.
   (c) The inverse operation for multiplying by \(-4\) is dividing by \(-4\).

3 Find at least three pairs of inverse operations for multiplication and for addition and show that the inverse element results.

4 The inverse operations for the sequence of operations \(+3, \ -5, \ \times2\), applied in an order to undo the operations, would be:
   \begin{align*}
   A & \quad -3, +5, \times2 \\
   B & \quad \times2, -5, +3 \\
   C & \quad -3, +5, +2 \\
   D & \quad +2, +5, -3 \\
   E & \quad +2, +5, +3
   \end{align*}

Applications

5 Dave has bought 3 tops each worth $45 and a pair of trousers for $69.
   (a) How much does he owe?
   (b) What would be the inverse operation to spending money?

Analysis

6 (a) Find the inverse of multiplying by 5.
   (b) Using multiplication, what would be the inverse of multiplying by 5?
   (c) Explain why the methods from (a) and (b) are equivalent.

11.4 Correspondence and graphing

Plotting ordered pairs on the Cartesian plane can be considered in terms of sets. The set of the \(x\)-values of the ordered pairs is called the **domain**. The \(x\)-values are then mapped onto the \(y\)-values. The set of the \(y\)-values is called the **range**. Looking at the following ordered pairs \((1, 2), (2, 4), (3, 5)\), it is possible to see that the set of \(x\)-values is \(\{1, 2, 3\}\), so the domain is \(x \in 1, 2, 3\). The \(y\)-values are \(\{2, 4, 5\}\), so the range is \(y \in 2, 4, 5\). This set of ordered pairs is said to have a one-to-one correspondence because each \(x\)-value has one \(y\)-value.

This is not always the case; look at the graph opposite showing the points \((-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\).

The domain for this set of ordered pairs is \(\{-2, -1, 0, 1, 2\}\) and the range is \(\{0, 1, 4\}\). You will notice that the range has less elements than the domain. This is because some of the \(x\)-values map onto the same \(y\)-value. This type of correspondence is called a many-to-one correspondence. Even when we graph relationships there is a correspondence between the variables that may be one-to-one or many-to-one. A **function** is a relationship that is either one-to-one or many-to-one.
The graph of the function \( y = x + 1 \) is shown below:

$$
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
\hline
x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\hline
y & 10 & 5 & 2 & 1 & 2 & 5 & 10 \\
\hline
\end{array}
$$

It is possible to see that for each \( x \)-value there is only one \( y \)-value so this is a one-to-one correspondence.

**worked example 6**

Find the domain and range for the following set of ordered pairs and determine the type of correspondence that exists.

\( \{(1, 3), (2, 6), (3, -2), (4, 4)\} \)

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write out the set of ( x )-values to determine the domain.</td>
<td>Domain = {1, 2, 3, 4}</td>
</tr>
<tr>
<td>2. Write out the set of the ( y )-values to determine the range.</td>
<td>Range = {-2, 3, 4, 6}</td>
</tr>
<tr>
<td>3. Determine the type of correspondence by looking at whether any of the ( y )-values are repeated.</td>
<td>This is a one-to-one correspondence.</td>
</tr>
</tbody>
</table>

**worked example 7**

Graph the function \( y = x^2 + 1 \) using a table of values with \( x \)-values from -3 to 3 and determine the type of correspondence that exists.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solution</th>
</tr>
</thead>
</table>
| 1. Substitute the \( x \)-values into the function to find the \( y \)-values. | \( x = -3 \) then \( y = (-3)^2 + 1 = 10 \)  
\( x = -2 \) then \( y = (-2)^2 + 1 = 5 \)  
\( x = -1 \) then \( y = (-1)^2 + 1 = 2 \)  
\( x = 0 \) then \( y = 0^2 + 1 = 1 \)  
\( x = 1 \) then \( y = 1^2 + 1 = 2 \)  
\( x = 2 \) then \( y = 2^2 + 1 = 5 \)  
\( x = 3 \) then \( y = 3^2 + 1 = 10 \) |

2. Place the values you found in a table of values.
Other relationships between \(x\) and \(y\) can be plotted in the same way using a table of values. Sometimes care needs to be taken in the values that are chosen to be used in the table. For example, the graph of \(xy = 10\) is different to any graph you have previously seen. If you substitute the value \(x = 0\), then \(y\) cannot be found because 0 multiplied by any value gives 0. It would be easier to rearrange this equation in the form \(y = \frac{10}{x}\); since it is not possible to divide by 0, \(x = 0\) would not be a suitable value. The following example will show suitable values to substitute.

**worked example 8**

Sketch the graph of \(xy = 4\) by using a table of values.

**Steps**

1. Rearrange the equation so that it is in the form \(y = \frac{4}{x}\), remembering that when \(x\) and \(y\) are together there is an implied multiplication sign between them and the inverse operation of multiplication is division.

2. Find the values of \(y\) by substituting the values of \(x\) and recognising that \(x\) cannot equal 0.

**Solution**

\[
xy = 4 \\
\therefore y = \frac{4}{x}
\]

\[
x = -2 \therefore y = \frac{4}{-2} = -2 \]
\[
x = -1 \therefore y = -4
\]
\[
x = -\frac{1}{2} \therefore y = \frac{4}{-\frac{1}{2}} = 4 \times -2 = -8
\]
\[
x = \frac{1}{2} \therefore y = \frac{4}{\frac{1}{2}} = 4 \times 2 = 8
\]
\[
x = 1 \therefore y = 4
\]
\[
x = 2 \therefore y = 2
\]
1. Find the domain and range for the following set of ordered pairs and determine the type of correspondence that exists.
\{(2, 4), (0, -6), (1, -5), (4, -6)\}

2. Graph the function \( y = x^2 - 4 \) using a table of values with \( x \)-values from -3 to 3 and determine the type of correspondence that exists.

3. Graph each of the following functions using a table of values with \( x \)-values from -3 to 3 and determine the types of correspondence that exist.
   - \( a) \ y = x^2 + 5 \)
   - \( b) \ y = x^2 - 2 \)
   - \( c) \ y = x^2 + 2 \)
   - \( d) \ y = x^2 + 3 \)
   - \( e) \ y = -x^2 \)
   - \( f) \ y = -x^2 - 1 \)

4. Sketch the graph of each of the following functions by using a table of values.
   - \( a) \ xy = 6 \)
   - \( b) \ xy = 1 \)
   - \( c) \ y = \frac{10}{x} \)

5. For each of the following graphs determine the domain and range.

6. Draw at least one graph that shows a one-to-one correspondence and at least one graph that shows a many-to-one correspondence.
7 Which of the following graphs is a many-to-one correspondence?

![Graphs A to E](image)

**Applications**

8 Sunny buys pizzas from a particular shop but he never buys the same number. He paid $12 when he bought one pizza, he paid $19 when he bought two pizzas and $26 for three pizzas.

(a) Make the number of pizzas the $x$-value and the cost the $y$-value and plot these points.

(b) Find the equation of the relationship.

(c) State what sort of correspondence exists.

(d) What is the domain for the function?

(e) What is the range for the function?

**Analysis**

9 The domain of a function is given by $0 \leq x \leq 10$, the function is defined as $y = x^2 + 1$.

(a) Use a table of values to plot the graph of the function.

(b) State the type of correspondence that exists with this limited domain.

(c) State the range of the function.

---

**11.5 Inequalities**

The symbols $<, \leq, >, \geq$ mean less than, less than or equal to, greater than, greater than or equal to, respectively, and you have used these before. In this section you are going to look at whether values satisfy certain conditions. An **inequality** is the name given to an equation that has one of these symbols instead of an equals sign. Given the inequality $x > y$, let’s find out if the ordered pair $(3, 3)$ satisfies this condition. To find the answer we substitute the $x$- and $y$-values into the inequality and check if it holds true.
y-values into the inequality and get $3 > 3$ (3 is greater than 3), which is clearly not correct since $3 = 3$. So the statement is false.

**worked example 9**

Determine whether each of the following ordered pairs satisfies the inequality $x^2 > 2y$

(a) $(3, 1)$  
(b) $(3.5, 6)$  
(c) $(-1, 2)$

**Steps**

(a) 1. Substitute the $x$-value and $y$-value into the inequality.
2. Simplify and check the result.

(b) 1. Substitute the $x$-value and $y$-value into the inequality.
2. Simplify and check the result.

(c) 1. Substitute the $x$-value and $y$-value into the inequality.
2. Simplify and check the result.

**Solutions**

(a) $3^2 > 2 \times 1$
   
   $9 > 2$
   
   This is true so $(3, 1)$ satisfies the inequality $x^2 > 2y$.

(b) $3.5^2 > 2 \times 6$
   
   $12.25 > 12$
   
   This is true so $(3.5, 6)$ satisfies the inequality $x^2 > 2y$.

(c) $(-1)^2 > 2 \times 2$
   
   $1 > 4$
   
   This is false so $(-1, 2)$ does not satisfy the inequality $x^2 > 2y$.

**exercise 11.5 Inequalities**

1 Determine whether each of the following ordered pairs satisfies the inequality $x^2 > y$.
   
   (a) $(2, 1)$  
   (b) $(1.5, 6)$  
   (c) $(0, 2)$

2 Determine whether the point $(0, 0)$ is on the line, above the line or below the line for each of the following lines:
   
   (a) $y = 2x$  
   (b) $y = x + 1$
   
   (c) $y = -2x + 1$
   
   (d) $y = -x$
   
   (e) $y = -2x - 4$
   
   (f) $y = x + 6$

3 A distance, $d$, needs to be less than the square of the height, $h$.
   
   (a) Write an inequality that shows this.
   
   (b) If the distance is 4 m and the height is 1.5 m, determine whether this condition is fulfilled.

4 Write an inequality that involves $x^2$. Find at least three coordinate points for which the inequality is true and at least three coordinate points for which the inequality is false.
For the inequality \( y < x^2 - 4 \) the point that doesn’t satisfy the condition is:

- A \((1, -6)\)
- B \((0, -4)\)
- C \((2, -1)\)
- D \((-1, -4)\)
- E \((1.2, -5)\)

**Applications**

The curve \( h = d^2 + 4 \) represents the distance, \( d \), from an object and the height, \( h \), of a rope that is hanging. Determine whether the symbol should be \(<\), \(=\) or \(>\) for each of the following values of \( h \) and \( d \).

(a) \( h = 5, d = 1 \)
(b) \( h = 2, d = 0 \)
(c) \( h = -5, d = 20 \)

**Analysis**

(a) Using a table of values draw the graph of \( y = x^2 + 5 \).

(b) Plot each of the following points on the graph: \((0, 0)\), \((1, 5)\), \((2, 10)\) and \((-1, 7)\).

(c) Use these points, by substituting them into the inequality, to help you shade the section of the graph that represents \( y > x^2 + 5 \).

**VELS design task**

**Checking for safety**

**Investigating and designing**

A wire runs from the top of a house from a position that is 4 m from the ground to the top of a 10 m pole that is 20 m away from the house. The ground between the house and the pole is perfectly flat.

1. Draw a diagram to illustrate the situation with the house on the left and the pole on the right.
2. On your diagram add \( x \)- and \( y \)-axes. The \( y \)-axis should be the side of the house and the \( x \)-axis will be the ground.

**Producing**

3. Find an equation for the line between the house and the pole.
4. What are the domain and range for the line?
5. A 3 m high truck needs to come to the house—at what point closest to the house can it get under the wire?
6. What is the height of the wire 10 m from the house?

**Analysing and evaluating**

7. If the pole is on the other side of the road to the house and the road is 14 m wide, what is the maximum height of any transport vehicle that can safely travel along the road?
Solve the following and arrange the letters in the order shown by the corresponding answers to find the cartoon caption.

For the sets $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5\}$, $\varepsilon$ (universal set) $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, find:

<table>
<thead>
<tr>
<th>N</th>
<th>R</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \cap B$</td>
<td>$A'$</td>
<td>$A \cup B'$</td>
</tr>
<tr>
<td>$U (A \cap B)'$</td>
<td>$P A \cup B$</td>
<td></td>
</tr>
</tbody>
</table>

Find the inverse operation for each of the following:

<table>
<thead>
<tr>
<th>M</th>
<th>E</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>adding 5</td>
<td>multiplying by 5</td>
<td>dividing by 5</td>
</tr>
</tbody>
</table>

Find the coordinates that fit the following conditions:

<table>
<thead>
<tr>
<th>T</th>
<th>H</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 2y - 4$</td>
<td>$y &gt; 3x^2$</td>
<td>$y &lt; 2x + 1$</td>
</tr>
</tbody>
</table>

\[
\begin{array}{|c|c|c|c|c|}
\hline
(0, 2) & (2, 15) & \div 5 & \emptyset & (0, 0) & \div 5 \\
\hline
\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} & -5 & (0, 0) & \emptyset & \{1, 3, 5, 7, 9\} \\
\hline
\{1, 2, 3, 4, 5, 6, 7, 8, 10\} & \times 5 & \{2, 4, 6, 7, 8, 9, 10\} & \emptyset & \div 5 \\
\hline
\end{array}
\]
DIY summary

Copy and complete the following, using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1. The _____________ of a set is the set that contains the values that are part of the _____________ that are not included in the original set.
2. The ___________ of two sets is all the elements of both sets combined.
3. The numbers 1, 2, 3 are both ___________ ___________ and ___________.
4. A _______ _______ uses a rectangular box to show the _____________ and circles to show the other sets.
5. A _________ is the set of all the possible ________s of a set.
6. The _________ _______ for addition is 0, and for multiplication 1.
7. A _________ is a relationship that can have a many-to-one correspondence or a one-to-one correspondence.

VELS personal learning activity

1. Draw a diagram with examples that helps you remember all the different types of numbers you have learnt about.
2. Make up a universal set, find two subsets and then draw a Venn diagram, showing all the information. Make sure that you label the intersection and the union of all the sets.
3. Draw two graphs: one that shows a one-to-one correspondence and one that shows a many-to-one correspondence. State the domain and range for each of the graphs you have drawn.

Skills

1. For ε = {a, b, c, d, e, f}, A = {a, e}, B = {a, b, c}
   (a) Draw a Venn diagram showing all the information.
   (b) Find A’
   (c) Find A ∩ B’
2 Draw a tree diagram to show all the results of tossing a die, followed by tossing a coin.

3 Amy, Jade and Vanessa like three boys: Tim, Marcus and Jye. The boy Amy likes doesn’t have three letters in his name. Vanessa isn’t interested in Tim. Use a truth table to find out which girl likes which boy.

4 Determine the inverse operation and the identity element for each of the following:
   (a) − 4
   (b) × 5
   (c) ÷ 5

5 Determine the type of correspondence that exists between the following coordinate points: (1, −3), (2, −3), (3, 0), (4, −5)

6 Find the domain and range for the coordinates from 5.

7 Sketch the graph of xy = 1 by using a table of values.

8 Determine whether each of the following coordinate points satisfies the condition $x^2 > 2y$.
   (a) (−2, 6)
   (b) (−1, 1)
   (c) (2, 2)

9 (a) Find the equation of the line shown in the following graph.

   ![Graph with line and table of values]

   (b) Find whether each of the following coordinate points is on the line, above the line or below the line.
   (i) (0, 0)  
   (ii) (−1, 5)  
   (iii) (1, 6)

Applications

10 A set contains all the possible results from rolling a die. Set $A$ contains all the even numbered results and set $B$ contains the numbers 1 and 2.
   (a) Draw a Venn diagram to show this situation.
   (b) Shade on the Venn diagram the area that represents $A \cup B$.
   (c) Find $A \cap B$.

11 Using the set of numbers {−2, 5, 6.3, 7.5, 100, $\sqrt{3}$, 9.3}
   (a) State which of the numbers are natural numbers.
   (b) State which of the numbers are real numbers.
12 Using the graph shown opposite:
   (a) Determine the domain and range.
   (b) State the type of correspondence.

13 (a) Using a table of values sketch the graph of \( y = 2x^2 \).
   (b) Plot the following points on the graph: (0, 1), (1, 5), (-2, 4).
   (c) Shade the area of the graph that would be represented by \( y > 2x^2 \).

Analysis

14 A student is investigating the popularity of sports and she asks a random group of students if they participate in winter or summer sport competitions. She surveys 90 students altogether and finds that 62 students play summer sport, 50 play winter sport and 18 play neither. Use a Venn diagram to find the number of students who:
   (a) play both summer and winter sports
   (b) play summer sport and not winter sport
   (c) do not play summer sport
   (d) play summer or winter sport but not both.

15 (a) Using a table of values sketch the graph of \( xy = 6 \).
   (b) State any value that \( x \) cannot be equal to and hence state the domain of the function.
   (c) State any value that \( y \) cannot be equal to and hence state the range of the function.
   (d) Determine what type of correspondence this function shows.
1 Evaluate without using a calculator.
   (a) \(-24 \div 4\)  
   (b) \(35 \div -7\)  
   (c) \(-50 \div -5\)  
   (d) \(-72 \div 6\)

2 Write each of the following in index form.
   (a) \(a \times b \times a \times b\)  
   (b) \(3 \times 3 \times 4 \times 3 \times 3\)  
   (c) \(4 \times t \times m \times t \times 4 \times m \times t\)

3 Calculate:
   (a) \(20\%\) of $56  
   (b) \(1\%\) of $90  
   (c) \(5.6\%\) of $120

4 Substitute \(a = -2\) and \(b = 1\) into the following expressions and then simplify.
   (a) \(2a(b + 1)\)  
   (b) \(3ab - 5\)  
   (c) \(\frac{a}{b} + 2b\)

5 Find the value of the pronumerals in each case.
   (a) \(a : 12 = 60 : 6\)  
   (b) \(4 : 7 = 100 : b\)  
   (c) \(1 : c = 3 : 22\)

6 A car travels 145 km in 2.1 hours. What is its average speed to the nearest km/h?

7 Solve each of the following equations.
   (a) \(\frac{3x + 2}{2} = 7\)  
   (b) \(\frac{a + 4}{3} = 2\)  
   (c) \(3(m + 1) = -9\)

8 Sketch the graph of the linear relationship \(y = 2x + 1\).

9 State the value of \(x^2\) in each of the following and state whether it is a corresponding, alternate or co-interior angle with the other angle given.

10 For the following data, draw an ordered stem-and-leaf plot, then find the median and interquartile range.
   14, 16, 26, 9, 15, 4, 16, 17, 24, 12, 11, 15

11 When rolling a normal die, find the following:
   (a) \(Pr\) (rolling an even number)  
   (b) \(Pr\) (not rolling a 1)  
   (c) \(Pr\) (rolling a 6)
DIY summary

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1. A decimal such as 2.63 is called a _______ _______.
2. We call the _______ of 5.
3. 5 is written in _______ _______ while 5 × 5 × 5 is written in _______ _______.
4. 67 is read as ‘six to the _______ of seven’.
5. Any decimal that has an infinite number of digits after the decimal point is called a _______ _______.
6. A comparison of one variable against another is sometimes called a _______.

VELS personal learning activity

Identify the mistakes that Jess has made in the following examples found in her workbook, and include the correct answers in your explanation.

1. 4 × 1 = 0.375
2. 1/3 as a decimal is 0.3
3. 2³ = 6
4. 1200 = 2 × 3 × 4 × 5
5. √5 = 25

Skills

1. Use your calculator to write the following improper fractions as (i) mixed numbers and (ii) decimals correct to two decimal places.
   (a) 752/93
   (b) 4921/75
   (c) 28643/104

2. The fraction 3797/11100, written in exact decimal form, is:
   A 342.07 B 0.34207 C 0.34207 D 0.34207 E 0.34207

3. Change the following recurring decimals into fraction form.
   (a) 0.2
   (b) 0.32
   (c) 0.32

4. Put the following into ascending order.
   (a) 0.283, 0.26
   (b) 0.22, 0.26, 0.253

5. Use your calculator to write the following mixed numbers as decimals.
   (a) 28\frac{3}{8}
   (b) 15\frac{3}{40}
   (c) 53\frac{261}{328}
**Hitting pay-dirt**

**Investigating and designing**

A number of people own an earth-moving business. Each person owns a percentage of the business. The operating capital is $375 000.

1. Work out the amount contributed by a person who has invested 7% of the operating capital.

   Profits are divided according to the percentage of the operating capital invested. For example, someone contributing 7% of the operating capital would receive 7% of the profits.

2. If the company makes $50 000, how much would the person who invested 7% receive?

3. Design an easy method (perhaps using a flowchart) that allows a part-owner to work out the total profit made by the company based on their share of the profits.

**Producing**

4. Use your method from 3 to work out the total profit if Janet, who has a 12% share, receives $3120.

5. Joshua contributed $82 500 to the operating capital of $375 000. Find out how much profit he makes.

6. If the rest of the operating capital is paid in equal amounts by 6 other people, how much profit does each of them make?

**Analysing and evaluating**

7. Use a different method to solve 6 and explain why you’d obtain the same answer.

8. Check your algorithm from 3 with your answer to 6. If you obtain different answers, work out the reason for the discrepancies.