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Welcome to Heinemann Maths Zone VELS Edition. This textbook and the accompanying student CD and teacher support material are part of an exciting new series that combines carefully crafted content written specifically for the Victorian Essential Learning Standards with a stunning, state-of-the-art, full colour design.

### Maths Zone VELS Edition textbook

#### VELS features

- VELS Personal Learning Activities that specifically address the Physical, Personal and Social Learning Strand through student self-reflection
- VELS Design Tasks that specifically address the Interdisciplinary Learning Strand using rich tasks
- VELS Assignments which integrate all three Strands.

#### Theory sections

- Concise, uncluttered explanations
- Clear diagrams and graphs
- Danger Zones highlighting common errors students should watch out for
- Worked Examples in Steps/Solutions format, which clearly separates instructions from working
- Highlight boxes providing concise summaries of major points

#### Home pages

- Motivating snippets of information related to the chapter
- Links to websites for further exploration

#### Exercises

- Exercises divided into Skills, Applications and Analysis questions
- Large numbers of carefully graded questions
- Explicit matches to Worked Examples
- Open-ended questions indicated by
- Multiple choice questions
- Bright, stimulating photos
**Prep Zones**

- Questions that help teachers ascertain if students have the prerequisite skills
- References to Replay worksheets, which provide remediation
- Key word list for mathematical literacy

**Maths in Actions and Maths@Works**

- Historical and real-life contexts for the mathematics being studied
- Weblinks to aid further research

**Investigations and Problem Solving tasks**

- Tasks related to the chapter content
- Group work icons where relevant
- Specific problem-solving strategies

**Computer, Graphics Calculator and CAS Investigations**

- Technology-based investigations
- Keystrokes for both TI and Casio models
- Computer Algebra Systems introduced

**Chapter Reviews**

- Do-it-yourself chapter summaries
- Skills, Applications and Analysis questions
- References back to relevant textbook sections
- Cumulative revision Replays

**Mixed revisions**

- Multi-chapter cumulative revision
- Mixed topic Skills, Applications and Analysis questions
- References back to relevant textbook sections
- Maths-competition style Challenges
Other Maths Zone features

• Key words bolded in text
• Answers with page references
• Glossary and index

Heinemann eMaths Zone VELS Edition student CD

Heinemann eMaths Zone VELS Edition is an interactive student CD that has been designed to complement the equivalent textbook. It contains the textbook in full, plus over 1000 dynamic links.

eTutorials

• Interactive multi-media presentations offering narrated alternative explanations and remedial support

eQuestions

• eQuestions providing extra on-screen practice

Animations

• Animations offering dynamic hints for specific questions
**eTesters**

- eTesters that generate random race-against-the-clock mental mathematics tests

**Interactives**

- Interactives that are randomly generated concept developers

**Hints**

- Pop-up hints that are included to provide help with specific questions

**Other Heinemann eMaths Zone VELS Edition features**

- Starter and Restarter problems providing an overarching context for chapters
  - Starter 3
  - Restarter 3

- Replay worksheets designed for remediation
  - Worksheet R1.4

- Consolidation worksheets providing students with extra practice
  - Worksheet C2.3

- Homework sheets specifically tailored to the textbook
  - Homework 5.1

- Chapter Assignments for test revision
  - Assignment 4

- Links to relevant websites
  - hi.com.au

- Bonus software
The Heinemann Maths Zone 10 VELS Edition Student Worked Solutions contains the worked solutions to every even-numbered question in the Heinemann Maths Zone 10 VELS Edition textbook plus the worked solutions to all the Prep Zones, Chapter Reviews and Mixed Revisions.


FlexiTest provides a bank of over 1000 fully editable questions tied to textbook sections, and an easy-to-use program that allows you to create tests or revision sheets and matching worked solutions at the click of a mouse.

The website www.hi.com.au/mathzonevels provides teachers with a complete package of VELS assessments and support, including:
- VELS Audits in editable form
- VELS Assessment Tasks and detailed assessment notes
- Solutions to Starters and Restarters, Homework Sheets, Chapter Assignments and Consolidation worksheets
- Teacher’s notes to VELS Design Tasks, VELS Assignments, Investigations, Problem Solving tasks, Maths in Actions, Maths@Works, DIY summaries and Challenge Maths.

Available for teachers is Heinemann Maths Zone 10 VELS Edition Teacher Worked Solutions, which contains the worked solutions to every question in the Heinemann Maths Zone 10 VELS Edition textbook, including Prep Zones, Maths in Actions, Maths@Works, Problem Solving tasks, Investigations, VELS Design Tasks, VELS Assignments, Chapter Reviews, DIY summaries and Challenge Maths.
Heinemann Maths Zone 9 and 10 VELS Edition VELS Level 6 Audit

The following Audit shows in detail where each of the Level 6 Victorian Essential Learning Standards for the Mathematics Discipline is covered in *Heinemann Maths Zone 9 VELS Edition* and *Heinemann Maths Zone 10 VELS Edition*. For other VELS-related material see www.hi.com.au/mathszonevels.

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<tr>
<th>VELS Mathematics Standards</th>
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<tbody>
<tr>
<td>Number</td>
<td></td>
</tr>
<tr>
<td>Students comprehend the set of real numbers containing natural, integer, rational and irrational numbers. They represent rational numbers in both fractional and decimal (terminating and infinite recurring) forms (for example, $1\frac{1}{3} = 1.15$, $0.\overline{6} = \frac{2}{3}$). They comprehend that irrational numbers have an infinite non-terminating decimal form. They specify decimal rational approximations for square roots of primes, rational numbers that are not perfect squares, the golden ratio $\varphi$, and simple fractions of $\pi$ correct to a required decimal place accuracy.</td>
<td>9 Ch 1 Ex 1.1, 1.3 10 Ch 2 Ex 2.1, 2.11</td>
</tr>
<tr>
<td>Students use the Euclidean division algorithm to find the greatest common divisor (highest common factor) of two natural numbers (for example, the greatest common divisor of 1071 and 1029 is 21, as $1071 = 1029 \times 1 + 42$, $1029 = 42 \times 24 + 21$ and $42 = 21 \times 2 + 0$).</td>
<td>9 Ch 1 Investigation: Euclidean division</td>
</tr>
<tr>
<td>Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms (one- or two-digit divisors in the case of division). They perform computations involving very large or very small numbers in scientific notation (for example, $0.0045 \times 0.000028 = 4.5 \times 10^{-3} \times 2.8 \times 10^{-5} = 1.26 \times 10^{-7}$).</td>
<td>9 Ch 1 Ex 1.1, 1.2, 1.3 10 Ch 2 Ex 2.1, 2.11</td>
</tr>
</tbody>
</table>
### VELS Mathematics Standards

<table>
<thead>
<tr>
<th>VELS Mathematics Standards</th>
<th>Heinemann Maths Zone 9 and 10 VELS Edition references</th>
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</thead>
<tbody>
<tr>
<td>They carry out exact arithmetic computations involving fractions and irrational numbers</td>
<td>9 Ch 1 Ex 1.1</td>
</tr>
<tr>
<td>(such as square roots (for example, $\sqrt{10} = 3.162$, $\sqrt{2} = 1.414$) and multiples</td>
<td>9 Ch 2 Ex 2.2</td>
</tr>
<tr>
<td>and fractions of $\pi$ (for example $\pi + \frac{5}{4} = 3.75$). They use appropriate</td>
<td>9 Ch 3 Ex 3.2, 3.3, 3.4, 3.5</td>
</tr>
<tr>
<td>estimates to evaluate the reasonableness of the results of calculations involving rational</td>
<td>10 Ch 2</td>
</tr>
<tr>
<td>and irrational numbers, and the decimal approximations for them. They carry out computations</td>
<td>10 Ch 8 Investigation: The unit circle</td>
</tr>
<tr>
<td>to a required accuracy in terms of decimal places and/or significant figures.</td>
<td></td>
</tr>
</tbody>
</table>

### Space

Students represent two- and three-dimensional shapes using lines, curves, polygons and circles. They make representations using perspective, isometric drawings, nets and computer-generated images. They recognise and describe boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra. They recognise the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties.

Students explore the properties of spheres.

<table>
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<tr>
<th>VELS Mathematics Standards</th>
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</thead>
<tbody>
<tr>
<td>Students represent two- and three-dimensional shapes using lines, curves, polygons and</td>
<td>9 Ch 2 Graphics calculator investigation: The</td>
</tr>
<tr>
<td>circles. They make representations using perspective, isometric drawings, nets and</td>
<td>properties of cylinders</td>
</tr>
<tr>
<td>computer-generated images. They recognise and describe boundaries, surfaces and</td>
<td>9 Ch 2 Ex 2.3, 2.4, 2.5, 2.6</td>
</tr>
<tr>
<td>interiors of common plane and three-dimensional shapes, including cylinders, spheres,</td>
<td>9 Ch 2 VELS Design Task: Do-it-yourself netball</td>
</tr>
<tr>
<td>cones, prisms and polyhedra. They recognise the features of circles (centre, radius,</td>
<td>court</td>
</tr>
<tr>
<td>diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use</td>
<td>9 Ch 8 Ex 8.8, 8.9</td>
</tr>
<tr>
<td>associated angle properties.</td>
<td>9 Ch 8 VELS Design Task: Electric fencing</td>
</tr>
<tr>
<td>Students explore the properties of spheres.</td>
<td>10 Ch 4 Ex 4.4, 4.6</td>
</tr>
<tr>
<td>Students explore the properties of spheres.</td>
<td>10 Ch 8 Ex 8.5, 8.6</td>
</tr>
<tr>
<td>VELS Mathematics Standards</td>
<td>Heinemann Maths Zone 9 and 10 VELS Edition references</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>Students use the conditions for shapes to be congruent or similar. They apply isometric</td>
<td>9 Ch 2 VELS Design Task: Do-it-yourself netball court</td>
</tr>
<tr>
<td>and similarity transformations of geometric shapes in the plane. They identify points</td>
<td>9 Ch 8 Ex 8.8, 8.9, 8.10, 8.11</td>
</tr>
<tr>
<td>that are invariant under a given transformation (for example, the point (2, 0) is</td>
<td>9 Ch 9 Ex 9.3</td>
</tr>
<tr>
<td>invariant under reflection in the y-axis, so the y-axis intercept of the graph of</td>
<td>10 Ch 4 Ex 4.6, 4.4, 4.5</td>
</tr>
<tr>
<td>y = 2x – 4 is also invariant under this transformation). They determine the effect</td>
<td>10 Ch 8 Maths in Action: The art of fractals</td>
</tr>
<tr>
<td>of changing the scale of one characteristic of two- and three-dimensional shapes</td>
<td>10 Ch 8 Ex 8.7</td>
</tr>
<tr>
<td>(for example, side length, area, volume and angle measure) on related characteristics.</td>
<td></td>
</tr>
<tr>
<td>They use latitude and longitude to locate places on the Earth’s surface and measure</td>
<td>10 Ch 8 Investigation: Great circles</td>
</tr>
<tr>
<td>distances between places using great circles.</td>
<td></td>
</tr>
<tr>
<td>Students describe and use the connections between objects/location/events according to</td>
<td>8 Ch 9 Ex 9.7, 9.8, 9.9</td>
</tr>
<tr>
<td>defined relationships (networks).</td>
<td></td>
</tr>
</tbody>
</table>
Measurement, chance and data

At Level 6, students estimate and measure length, area, surface area, mass, volume, capacity and angle. They select and use appropriate units, converting between units as required. They calculate constant rates such as the density of substances (that is, mass in relation to volume), concentration of fluids, average speed and pollution levels in the atmosphere. Students decide on acceptable or tolerable levels of error in a given situation. They interpret and use mensuration formulae for calculating the perimeter, surface area and volume of familiar two-and three-dimensional shapes and simple composites of these shapes. Students use Pythagoras’ theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Students estimate probabilities based on data (experiments, surveys, samples, simulations) and assign and justify subjective probabilities in familiar situations. They list event spaces (for combinations of up to three events) by lists, grids, tree diagrams, Venn diagrams, and Karnaugh maps (two-way tables). They calculate probabilities for complementary, mutually exclusive, and compound events (defined using and, or and not). They classify events as dependent or independent.
## VELS Mathematics Standards

<table>
<thead>
<tr>
<th>Property</th>
<th>Heinemann Maths Zone 9 and 10 VELS Edition references</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students comprehend the difference between a population and a sample.</td>
<td>9 Ch 7</td>
</tr>
<tr>
<td>They generate data using surveys, experiments and sampling procedures.</td>
<td>9 Ch 7</td>
</tr>
<tr>
<td>They calculate summary statistics for centrality (mode, median and mean), spread (boxplot, interquartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot). They distinguish informally between association and causal relationship in bivariate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables.</td>
<td>10 Ch 7</td>
</tr>
</tbody>
</table>

## Structure

<table>
<thead>
<tr>
<th>Property</th>
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<tbody>
<tr>
<td>Students classify and describe the properties of the real number system and the subsets of rational and irrational numbers. They identify subsets of these as discrete or continuous, finite or infinite and provide examples of their elements and apply these to functions and relations and the solution of related equations.</td>
<td>9 Ch 3 Ex 3.2</td>
</tr>
<tr>
<td>10 Ch 1 Ex 1.4</td>
<td></td>
</tr>
<tr>
<td>10 Ch 2 Ex 2.1</td>
<td></td>
</tr>
<tr>
<td>Students express relations between sets using membership, (\in), complement, (\prime), intersection, (\cap), union, (\cup), and subset, (\subseteq), for up to three sets. They represent a universal set as the disjoint union of intersections of up to three sets and their complements, and illustrate this using a tree diagram, Venn diagram or Karnaugh map.</td>
<td>10 Ch 1 Ex 1.4</td>
</tr>
<tr>
<td>10 Ch 10 Ex 10.1, 10.2, 10.3</td>
<td></td>
</tr>
</tbody>
</table>
### Students form and test mathematical conjectures, for example, 'What relationship holds between the lengths of the three sides of a triangle?'

This task is detailed in the Heinemann Maths Zone 9 and 10 VELS Edition references. For instance, students might investigate the properties of triangles and cylinders using software tools.

#### VELS Design Tasks
- **Maths in Actions**
  - **9 Ch 2 Graphics calculator investigation:** The properties of cylinders
  - **9 Ch 8 Computer investigation:** Congruent and similar triangles
  - **10 Ch 8 Computer investigation:** Geometrical conjectures
  - **10 Ch 8 Investigation:** Points on polygons and circles
  - **10 Ch 9 Graphics calculator investigation:** Investigating quadratics—how varying $b$ affects the graph of $y = ax^2 + bx + c$

### They use irrational numbers such as $\pi$, $\phi$, and common surds in calculations in both exact and approximate form.

This task is also detailed in the Heinemann Maths Zone 9 and 10 VELS Edition references. Students should be able to manipulate these numbers within calculations.

#### Heinemann Maths Zone 9 and 10 VELS Edition references
- **9 Ch 3**
- **9 Ch 2 Graphics calculator investigation:** The properties of cylinders
- **9 Ch 2 Ex 2.3**

### Students apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulae, rearrange and simplify algebraic expressions involving real variables. They verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal). (for example, $4x - 8 = 2(2x - 4)$; $2a - 3b = 4a^2 - 12a + 9$;

$(m^3)^2 = 27w^2; \frac{2x^2}{xy^2} = x^2y^{-1}; \frac{4}{xy} x = \frac{2}{y}$)

This task is also detailed in the Heinemann Maths Zone 9 and 10 VELS Edition references. Students should be able to apply these properties to solve problems involving algebraic expressions.

#### Heinemann Maths Zone 9 and 10 VELS Edition references
- **9 Ch 1 Ex 1.4**
- **9 Ch 4**
- **10 Ch 1 Ex 1.5**
- **10 Ch 2 Ex 2.2, 2.5, 2.7, 2.8, 2.9, 2.12, 2.14**
- **10 Ch 3 Ex 3.9**
Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the Cartesian coordinate system) with consideration of independent and dependent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ratio between consecutive terms (for example to distinguish between the functions described by the sets of ordered pairs {(1, 2), (2, 4), (3, 6), (4, 8) ...}; {(1, 2), (2, 4), (3, 8), (4, 14) ...}; and {(1, 2), (2, 4), (3, 8), (4, 16) ...}). They use and interpret the functions in modelling a range of contexts.

They recognise and explain the roles of the relevant constants in the relationships \( f(x) = ax + c \), with reference to gradient and y-axis intercept, \( f(x) = a(x + b)^2 + c \) and \( f(x) = ca^x \).

They solve equations of the form \( f(x) = k \), where \( k \) is a real constant (for example, \( x(x + 5) = 100 \)) and simultaneous linear equations in two variables (for example \( 2x - 3y = -4 \) and \( 5x + 6y = 27 \)) using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods.
### Working mathematically

Students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, ‘if $m^2$ is even then $m$ is even, and if $m^2$ is odd then $m$ is odd’). They follow formal mathematical arguments for the truth of propositions (for example, ‘the sum of three consecutive natural numbers is divisible by 3’).

<table>
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| Students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, ‘if $m^2$ is even then $m$ is even, and if $m^2$ is odd then $m$ is odd’). They follow formal mathematical arguments for the truth of propositions (for example, ‘the sum of three consecutive natural numbers is divisible by 3’). | 9 Ch 6 Investigation: How to find the 1,000,000th term in a number pattern in a few seconds  
10 Ch 1 Investigation: The ‘hamburger index’  
10 Ch 9 Investigation: A pendulum experiment |

Students choose, use and develop mathematical models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts (for example, exact and approximate measurement formulae for the volumes of various three-dimensional objects such as truncated pyramids). They generalise from one situation to another, and investigate it further by changing the initial constraints or other boundary conditions. They judge the reasonableness of their results based on the context under consideration.

| Home page Starters  
VELS Design Tasks  
Maths in Actions  
Maths@Works  
9 Ch 1 Graphics calculator investigation: Using iteration to investigate savings plans  
9 Ch 2 Graphics calculator investigation: The properties of cylinders  
9 Ch 4 Investigation: Increasing profit  
9 Ch 5 Investigation: Viewing artwork—where should you stand?  
9 Ch 7 CAS Investigation: Making predictions  
9 Ch 8 Investigation: Drawing Escher-style pictures  
10 Ch 2 Investigation: The von Koch snowflake  
10 Ch 4 Investigation: Volumes of tapered solids  
10 Ch 10 Graphics calculator investigation: Investigating the reliability of lie detectors  
10 Ch 10 Investigation: Fingerprint probabilities |
They select and use technology in various combinations to assist in mathematical inquiry, to manipulate and represent data, to analyse functions and carry out symbolic manipulation. They use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

<table>
<thead>
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<tbody>
<tr>
<td>9 Ch 1 Graphics calculator investigation: Using iteration to investigate savings plans</td>
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<td>9 Ch 2 Graphics calculator investigation: The properties of cylinders</td>
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</tr>
<tr>
<td>10 Ch 2 CAS investigation: Surds made simple</td>
<td>10 Ch 2 CAS investigation: Surds made simple</td>
</tr>
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<td>10 Ch 3 Graphics calculator investigation: Checking expansions</td>
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<td>10 Ch 4 Graphics calculator investigation: A manufacturing problem</td>
<td>10 Ch 5 CAS investigation: Equation of a tangent</td>
</tr>
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<td>10 Ch 5 Graphics calculator investigation: The art of linear relationships</td>
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<tr>
<td>10 Ch 8 Computer investigation: Geometrical conjectures</td>
<td>10 Ch 9 Computer investigation: Solving quadratics using iteration</td>
</tr>
<tr>
<td>10 Ch 9 Graphics calculator investigation: Investigating quadratics—how varying ( b ) affects the graph of ( y = ax^2 + bx + c )</td>
<td>10 Ch 9 Graphics calculator investigation: Investigating the reliability of lie detectors</td>
</tr>
<tr>
<td>10 Ch 9 CAS investigation: Polar graphs</td>
<td>10 Ch 10 Graphics calculator investigation: Investigating the reliability of lie detectors</td>
</tr>
</tbody>
</table>
VELS Assignments

Assignment 1: How much is all right?

Chapter references: 2 and 5

You will need: graph paper

The price of a particular item in various sizes can vary. A fair assumption is that if you buy in bulk, then it is possible to get goods at a cheaper relative price.

1. Determine at least three items that you would like to investigate the relative price of. These may be soft drink, flour, jam, Vegemite, chips, confectionery or anything else that comes in a variety of sizes.

2. Find the price and volume or weight of each of the items in as many different sizes as possible. Also, comment on any differences in the type of packaging for the different sizes.

3. For each item, plot the points with the weight/volume on the horizontal axis and the price on the vertical axis. Draw a line of best fit and determine its equation.

4. Is it true that the price of an item increases at the same rate as its weight/size? If not, is it always cheaper to buy the bigger product?

5. What other factors do you think affect the price of an item?
Assignment 2: Take a guess
Chapter references: 4
You will need: A variety of containers of different shapes, a variety of locations around the school, a measuring cylinder, a tape measure, a thermometer and a calculator

1. Draw up a table such as the following.

<table>
<thead>
<tr>
<th>Measurement description</th>
<th>Guess</th>
<th>Estimate with more information</th>
<th>Actual measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Volume of container 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Volume of container 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Volume of container 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Distance from back of classroom to front</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Height of the door</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Area of the classroom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Area of the oval</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. Temperature in the classroom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Temperature in the shade outside</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Temperature in the sun outside</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Fill in the table without discussing your ideas with anyone. Make sure that you include units.

3. For each of the ten items, write down what you think would be helpful to increase the accuracy of your estimate; e.g. looking at the oval might make your estimate of its area more realistic.

4. Using the assistance provided, fill in the third column of the table.

5. Take the appropriate measurements and find the actual figures.

6. Determine where the greatest errors occurred and what assisted most in making the estimate most accurate.

Assignment 3: Our world
Chapter references: 1, 7 and 9
You will need: Internet access and graphics calculator
Countries around the world have different features. These include the country’s area, the number of people, the average income of people in the country and foreign debt.

1. Using Australia and at least fifteen other countries, find out some statistics about each country. You should at least find out the area, population and average income.

2. Using your graphics calculator and the instructions on page 374, produce a boxplot to show the variation in population and average income.
According to a court-appointed accountant, Celia’s original deposit of US$24,000 in February 1988, compounded to US$46 billion in late 1999 and then US$2.3 trillion by the time she won her court ruling and appeal. Sky-high interest rates in Mexico during the 1990s, peaking at 149.35%, pushed the monthly compounding figure she was owed past the total of Mexico’s entire foreign currency reserves. When the ruling faced a final appeal before the Supreme Court, Celia’s lawyers suggested they would negotiate a realistic settlement.
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your eMaths Zone CD or ask your teacher for the Replay Worksheet.

1. Express:
   (a) \( \frac{2}{13} \) as a decimal, correct to three decimal places
   (b) 35% as a fraction, in simplest terms
   (c) \( \frac{2}{5} \) as a percentage, correct to two decimal places
   (d) 0.384 as a fraction, in simplest terms
   (e) \( \frac{2}{5} \) as a recurring decimal.

2. (a) Calculate the commission on a house sold for $175 000 if the real estate agent is paid 1.5% on the first $90 000 and 1.25% on the rest.
   (b) Calculate the commission on a house sold for $450 000 if the real estate agent is paid 1.75% on the first $150 000 and 1.45% on the rest.

3. (a) A painter is allowed a discount of 30% on bulk paints. How much will a $55 can of paint cost the painter?
   (b) A builder is allowed a discount of 25% on timber supplies. How much will $8400 of timber cost the builder?

4. (a) A pair of socks is sold for $8. What is the cost price if the 28% profit was calculated on the cost price?
   (b) A jumper is sold for $60. What is the cost price if the 20% profit was calculated on the cost price?

5. For the set \( A = \{2, 4, 6, 8\} \) \( B = \{3, 4, 5\} \) find:
   (a) \( A \cap B \)
   (b) \( A \cup B \).

**KEY WORDS**

- complement
- discrete
- effective profit
- element
- empirical induction
- finite set
- induction
- infinite set
- inflation
- intersection
- Karnaugh map
- mathematical induction
- union
- universal set

Sample pages only
Many bills to pay are associated with living in a house, unit or apartment. These include the telephone, electricity, gas, water rates and local government rates. Many mathematical aspects are involved in each of these types of bill. Some of these bills (e.g. gas and electricity) now provide a graphic representation of average daily energy usage that compares the current billing period to previous billing periods.
Skills

1. From the information shown on the Loki Energy bill on page 3, find the total bill for the billing period 4 February to 5 April 2003, assuming all costs remained at the same level.
Sparx Power supplies electricity to domestic households. The amount of electric energy used is measured by the unit *kilowatt hour* (kWh). Sparx Power’s 2003 tariff chart is as follows. Bills are issued each calendar month.

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Units (kWh)</th>
<th>Cents/kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic light</strong></td>
<td>First 100 kWh per residence per month</td>
<td>15.14</td>
</tr>
<tr>
<td>and power</td>
<td>Next 300 kWh per residence per month</td>
<td>10.29</td>
</tr>
<tr>
<td><strong>Tariff 11</strong></td>
<td>Remainder</td>
<td>9.18</td>
</tr>
<tr>
<td></td>
<td>Minimum payment per residence per month $6.80</td>
<td></td>
</tr>
<tr>
<td><strong>Super economy</strong></td>
<td>All consumption</td>
<td></td>
</tr>
<tr>
<td><strong>Night rate</strong></td>
<td><em>This tariff is used for hot water services that heat up at night, or</em></td>
<td></td>
</tr>
<tr>
<td><strong>Tariff 31</strong></td>
<td><em>for a variety of other permanently connected loads, including</em></td>
<td>4.25</td>
</tr>
<tr>
<td></td>
<td><em>heatbanks. Electricity supply is restricted to 8 hours per day.</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum monthly payment per residence $2.80</td>
<td></td>
</tr>
<tr>
<td><strong>Controlled</strong></td>
<td>All consumption</td>
<td>6.22</td>
</tr>
<tr>
<td><strong>supply</strong></td>
<td><em>This tariff can be applied to most electric storage water heaters,</em></td>
<td></td>
</tr>
<tr>
<td><strong>Tariff 33</strong></td>
<td><em>solar–electric water heaters and heat pump water heaters.</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>Electricity supply is available for at least 18 hours per day and is</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>switched via load control equipment supplied and maintained by</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>the company.</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Minimum monthly payment per residence $2.80</td>
<td></td>
</tr>
</tbody>
</table>

Find the cost of the following monthly quantities of electricity for a domestic household.

(a) 450 kWh on tariff 31
(b) 600 kWh on tariff 31
(c) 800 kWh on tariff 33
(d) 490 kWh on tariff 33
(e) 740 kWh on tariff 11
(f) 1000 kWh on tariff 11
3 (a) For Sparx Power, find the number of kWh of electricity equivalent in cost to the minimum monthly payment for:
   (i) tariff 31   (ii) tariff 11.
(b) Find the percentage saving, compared to tariff 11, on 500 kWh for:
   (i) tariff 33   (ii) tariff 31.

4 The following table shows the call costs associated with timed local calls for a telephone company.

<table>
<thead>
<tr>
<th>Call type</th>
<th>Charge (cents)</th>
<th>Chargeable period (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Day rate 8 a.m.–6 p.m. Mon.–Fri.</td>
</tr>
<tr>
<td>Local call</td>
<td>18.0</td>
<td>First 4 minutes</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>Each 1 minute thereafter</td>
</tr>
</tbody>
</table>

Note: As soon as you start a new period you must pay for the whole period.

(a) Find the cost of the following local calls (in dollars).
   (i) 16 minutes at 10 a.m. Monday
   (ii) 24 minutes at 9 p.m. Thursday
   (iii) 20 minutes at 11 p.m. Friday
   (iv) 22 minutes at 7 a.m. Sunday
   (v) 35 minutes at 2 p.m. Tuesday
(b) Find the percentage difference in costs for each of these pairs of local calls.
   (i) 12 minutes at day rate, 12 minutes at night rate
   (ii) 21 minutes at day rate, 21 minutes at night rate
   (iii) 7 minutes at day rate, 7 minutes at night rate
   (iv) 4 minutes at day rate, 4 minutes at night rate
(c) The company claims that night rate and economy rate, which are the same for local calls, can result in savings of up to 47% off the day rate. What do you think about this claim?

5 Electricity use is charged at the following rates:
   12.27 cents per kWh for the first 1020 kWh
   12.93 cents per kWh for the remaining use

(a) The cost for 500 kWh of electricity use is closest to:
   A $64.65   B $125.15   C $131.89   D $61.35   E $67.32
(b) The cost for 1700 kWh of electricity use is closest to:
   A $208.59   B $213.07   C $219.81   D $215.33   E $87.92
(c) The cost for 2200 kWh of electricity use is closest to:
   A $276.68   B $269.94   C $284.46   D $145.20   E $277.72
Applications

6 (a) From the information shown on the Loki Energy bill on page 3, find the total bill for the billing period 6 April to 5 June 2003, assuming all costs remained at the same level.
(b) The bill indicates the actual amount paid for this period. How much was it?
(c) How do your answers to parts (a) and (b) compare? Can you explain any differences?

7 A 60 W globe in a light fitting uses 60 Wh of electricity every hour.
(a) How many kWh of electricity would such a globe use in a three-month period (say 92 days) if it was on for an average of 2.5 hours per day?
(b) How much would this cost if electricity was charged at 14.11 cents per kWh?
(c) How much would you save if you replaced the globe with a 40 W one?

8 Think about the light globes that you have in your house and the average use for each per day. Calculate the cost of running them if the charge is 14.11 cents per kWh.

Analysis

9 Many people have a mobile telephone. The following table compares costs associated with plans available from a number of different companies. Each plan is designed for the medium use consumer who would make between 15 and 30 calls per week. Each plan has a 24-month contract, except for Orange Mobile which has no contract. For each plan the billing increment is 30 seconds. This means that a 31-second call costs the same as a 60-second call.

<table>
<thead>
<tr>
<th>Service provider</th>
<th>Plan name</th>
<th>Free calls included</th>
<th>Minimum monthly charge</th>
<th>Flagfall Peak 30 sec</th>
<th>Flagfall Off-peak 30 sec</th>
<th>Fixed line rates Peak 30 sec</th>
<th>Fixed line rates Off-peak 30 sec</th>
<th>Mobile to mobile Peak 30 sec</th>
<th>Mobile to mobile Off-peak 30 sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aussie Phones</td>
<td>True Blue Plus 65</td>
<td>$55.00</td>
<td>$65.00</td>
<td>16.5c</td>
<td>16.5c</td>
<td>22c</td>
<td>22c</td>
<td>22c</td>
<td>22c</td>
</tr>
<tr>
<td>B Clear and Simple</td>
<td>B Active 59</td>
<td>$59.00</td>
<td>$59.00</td>
<td>25.3c</td>
<td>25.3c</td>
<td>29.7c</td>
<td>22c</td>
<td>29.7c</td>
<td>22c</td>
</tr>
<tr>
<td>New Tel</td>
<td>Hear It 47</td>
<td>$47.00</td>
<td>$47.00</td>
<td>17c</td>
<td>17c</td>
<td>28c</td>
<td>28c</td>
<td>28c</td>
<td>28c</td>
</tr>
<tr>
<td>Optus</td>
<td>Your Call 55</td>
<td>$55.00</td>
<td>$55.00</td>
<td>22c</td>
<td>22c</td>
<td>26.4c</td>
<td>26.4c</td>
<td>26.4c</td>
<td>26.4c</td>
</tr>
<tr>
<td>Orange Mobile</td>
<td>AnyTime 45</td>
<td>$45.00</td>
<td>$45.00</td>
<td>22c</td>
<td>22c</td>
<td>30c for first 10 min, then 18c/30 sec (no flagfall). 18c/30 sec to other networks</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Find the cost of the following Peak calls for each network. (For Orange Mobile, assume that the call is to another network.)
   (i) 20 seconds  (ii) 55 seconds
   (iii) 130 seconds  (iv) 250 seconds
(b) Find the costs for the same calls at Off-peak rates.
(c) How is the Aussie Phones plan different from the others with regard to the minimum monthly cost?
(d) Make a list of the things you would need to consider when deciding which plan would be best for you.
Energy from the wind

Since the middle of 2002, some Victorian households have been able to purchase green electricity as part of their overall electricity package. For some of these consumers the source of this green electricity has been the wind.

There are two large-scale wind farms in Victoria: Codrington, near Portland in western Victoria, which is capable of producing 18.2 MW (megawatts), and Toora in South Gippsland, which can produce 21 MW. In addition, there is a single turbine generator at Breamlea, near Geelong. In fact, the turbine at Breamlea was one of the first installed in Australia and was part of the testing procedures to see how efficient the technology might be. It produces just 0.06 MW of power.

Wind energy in Victoria replaces power that would otherwise be generated by coal-fired plants. As a result there is a substantial benefit to the community in terms of a reduction in the emission of greenhouse gases. It has been estimated that each kWh of coal-fired electricity production emits 860 g of carbon dioxide (CO₂), 10 g of sulphur dioxide (SO₂) and 3 g of nitrous oxides (NOₓ). It should be noted that the amount of greenhouse gases emitted by coal-fired generators is falling all the time, due to increases in the efficiency of the equipment and the installation of pollution filters.
The reduction in gas emissions can be calculated using the following formulae:

\[
\text{CO}_2 \text{ (in tonnes)} = \frac{A \times 0.3 \times 8760 \times 860}{1000}
\]

\[
\text{SO}_2 \text{ (in tonnes)} = \frac{A \times 0.3 \times 8760 \times 10}{1000}
\]

\[
\text{NO}_x \text{ (in tonnes)} = \frac{A \times 0.3 \times 8760 \times 3}{1000}
\]

where \( A \) = the rated capacity of the turbine in MW;

0.3 is a constant, the capacity factor, that takes into account the variable nature of the wind and energy losses from the array; and

8760 is the number of hours in a year.

It has been estimated that the Codrington site will save up to 71 000 tonnes of greenhouse gas emissions each year. This is the equivalent of taking 17 000 cars off the road each year. The electricity produced, about 55 000 MWh, will be enough to supply 11 000 homes. The Toora site is expected to provide power for 6600 homes and save 48 000 tonnes of greenhouse gas emissions per year.

It is interesting to compare coal-fired generators to wind turbines. It has been estimated that the average wind farm will repay the energy used in its construction within six months. When fuel is taken into account, coal-fired generators never pay back the total energy cost. Also, as we know, wind is a renewable energy source—it will never run out.

As well as producing clean electricity, wind farms have also proven to be a boost for the local economy through tourism increasing. In just one year 70 000 people have entered the Codrington wind farm site. This confirms data from other Australian and overseas wind farms.

Although the Codrington wind farm is set on a 240 ha site, the actual turbines take up less than 1% of the site. The rest can still be used as a grazing property.

**Questions**

1. Calculate the reduction in each gas due to the use of a wind turbine with a rated capacity of 0.66 MW.

2. Based on the figures given for Codrington, what is the average electricity consumption of a Victorian home?

3. Use this number to calculate the number of MWh of electricity expected to be produced annually by the Toora site.

4. By keeping the ratio of gas emissions the same, calculate the \( A \) value (rated capacity) for both the Codrington and Toora sites.

5. Why do you think there have been objections to the construction of wind turbines when they seem like such a great source of energy production?

**Research**

1. Find out about the Portland Wind Energy Project. Prepare a PowerPoint presentation about the project including information that supports the project as well as highlighting some of the concerns that exist with regard to this type of energy production.

2. What other sources of green electricity exist? Construct a poster showing some of the alternatives available or prepare a short talk for your class, with some visual aids.
However, when money is borrowed—as in the case of a personal loan or a house mortgage—and the interest is added (compounded) at regular intervals to the amount owing, thus increasing the amount owed in the next period, we are dealing with compound interest. Compound interest also applies to investment, especially when the money is invested for a fixed period.

Compound interest calculations can be done as a series of simple interest calculations with the interest being added to the principal after each of the interest periods.

Note that in the simple interest case above the amount of interest payable each year is a constant $1800, whereas in the compound interest case it increases each year because of the interest being added to the principal. In other words, interest is earning even more interest.

### Worked example 2

Clive borrows $12 000 to buy a lathe for his factory. He agrees to pay 15% interest p.a. and to repay the debt in full in three years’ time. Calculate:

(a) the amount to be repaid if interest is compounded annually

(b) the amount to be repaid if simple interest is charged.

### Steps

(a) 1. Write the formula for simple interest.

   \[ P = 12 000 \]

   \[ R = \frac{15}{100} = 0.15, \ T = 1 \]

   \[ SI = 12 000 \times 0.15 \times 1 = $1800 \]

   3. Identify \( P \) for year 1.

   \[ P = 12 000 + 1800 = 13 800 \]

   6. Substitute (\( R \) and \( T \) unchanged).

   \[ SI = 13 800 \times 0.15 \times 1 = $2070 \]

   7. Calculate \( P \) for year 3.

   \[ P = 13 800 + 2070 = 15 870 \]

   8. Substitute (\( R \) and \( T \) unchanged).

   \[ SI = 15 870 \times 0.15 \times 1 = $2380.50 \]

   9. Calculate amount owed.

   \[ Total \ owed = 15 870 + 2380.50 = $18 250.50 \]

(b) 1. Write the formula.

   \[ SI = PRT \]

   2. Identify \( P \).

   \[ P = 12 000 \]

   3. Identify \( R \) and \( T \) (total time).

   \[ R = 0.15, \ T = 3 \]

   \[ SI = 12 000 \times 0.15 \times 3 = $5400 \]

   5. Calculate amount owed.

   \[ Total \ owed = 12 000 + 5400 = $17 400 \]

Note that in the simple interest case above the amount of interest payable each year is a constant $1800, whereas in the compound interest case it increases each year because of the interest being added to the principal. In other words, interest is earning even more interest.
When principal, rate and time are the same, compound interest will always be more than simple interest, except that if there is only one interest period they will be equal.

The compound interest calculation can be set up in a spreadsheet with the first column showing the year, the second column showing principal and the third column showing the interest. The required formulae are shown at right.

The actual values would then be as shown at right, which is the result obtained in Worked Example 2.

As calculations on compound interest are tedious even if done with a computer spreadsheet program, it would be preferable to establish a formula.

To increase a number by a percentage, we multiply the original number by 1 plus the percentage. For example, to increase 500 by 10% we do

\[ 500 \times (1 + 10\%) = 500 \times 1.1 \]

Compound interest requires a series of these calculations to occur as interest is calculated and added on each period.

In general, the total amount owing can be written as:

\[ A = P(1 + i)^n \]

where

\[ i = \frac{\text{interest rate per annum}}{\text{number of periods per annum}} = \frac{R}{n} \]

which is expressed as a decimal

and

\[ n = \text{number of years} \times \text{number of periods per year} = T \times \text{number of periods per year} \]

This form of the formula can be used whether the compounding period is a year or otherwise.

The compound interest formula is: \[ A = P(1 + i)^n \]

where \[ P = \text{amount borrowed} \]

\[ i = \text{rate of interest per period as a decimal} \]

and \[ n = \text{number of interest periods} \]

Note that \[ n \] is not necessarily \( T \), the length of the loan in years.

Thus, if interest of 12% p.a. is paid each three months for five years, the interest rate for each period of three months is

\[ i = \frac{12\%}{4} = 3\% = 0.03 \]

and the number of periods is \[ n = 5 \times 4 = 20. \]
In Worked Example 2, Clive borrowed $12,000 to buy a lathe for his factory. He agreed to pay 15% interest p.a. to be repaid in full in three years' time. Calculate the amount to be repaid if the interest is compounded:

(a) annually  
(b) half-yearly  
(c) quarterly.

**Steps**

(a) 1. Write the compound interest formula.
2. Identify $P$.
3. Identify $i$ and $n$.
4. Substitute.
5. Calculate $1 + 0.15$ first, then use $x^y$ on your calculator (some calculators may have $a^x$ or $y^x$) to calculate $1.15^3$.

(b) 1. Write the formula.
2. Identify $P$.
3. Interest is compounded every half-year, so there are two interest periods per year. Calculate $i$.
4. Calculate $n$.
5. Substitute and calculate $A$.

(c) 1. Write the formula.
2. Identify $P$.
3. Interest is compounded every quarter-year, so there are four interest periods per year. Calculate $i$.
4. Calculate $n$.
5. Substitute.
6. Evaluate.

**Solutions**

(a) $A = P(1 + i)^n$

$P = 12,000$

$i = 0.15, n = 3$

$A = 12,000(1 + 0.15)^3$

$= 18,250.50$

(b) $A = P(1 + i)^n$

$P = 12,000$

$i = \frac{0.15}{2}$

$= 0.075$

$A = 12,000(1 + 0.075)^6$

$= 18,519.62$

(c) $A = P(1 + i)^n$

$P = 12,000$

$i = \frac{0.15}{4}$

$= 0.0375$

$A = 12,000(1 + 0.0375)^{12}$

$= 18,665.45$

Assuming that all other things are kept the same, the more frequently interest is calculated, the greater the total will be.
Applications of the compound interest formula

Quite often, three of the variables used in the compound interest formula are known and the fourth needs to be found.

worked example 4

(a) Aunt Freda leaves Thelma a legacy—some deposit stock that was invested for ten years at 11% p.a. compounded quarterly. The value of the cheque received was $38 478.36. Calculate the initial deposit.

(b) When she first began it, Claudia's business had takings of $228 000 p.a. Her takings twelve years later are $520 000. At what rate p.a. (correct to one decimal place) is her business growing?

(c) After how many years will a $450 stamp be worth at least $900 if it increases in value by 7.5% p.a.?

Steps

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 1. Write the formula.</td>
<td>(a) [ A = P(1 + i)^n ]</td>
</tr>
<tr>
<td>2. Identify ( P ), unknown.</td>
<td>( P = ? )</td>
</tr>
<tr>
<td>3. Calculate ( i ) and ( n ).</td>
<td>( i = \frac{0.11}{4} = 0.0275 )</td>
</tr>
<tr>
<td>4. Identify ( A ).</td>
<td>( A = 38 478.36 )</td>
</tr>
<tr>
<td>5. Substitute.</td>
<td>( 38 478.36 = P(1 + 0.0275)^{40} )</td>
</tr>
<tr>
<td>6. Calculate ((1 + 0.0275)^{40}) using your calculator.</td>
<td>( 38 478.36 = 2.959 873 987 P )</td>
</tr>
<tr>
<td>7. Transpose to find ( P ).</td>
<td>( P = \frac{38 478.36}{2.959 873 987} = \frac{38 478.36}{2.959 873 987} = 13 000 )</td>
</tr>
<tr>
<td>8. Evaluate.</td>
<td>( \approx $13 000 )</td>
</tr>
</tbody>
</table>

(b) 1. Write the formula. \[ A = P(1 + i)^n \]

2. Identify \( P \). \( P = 228 000 \)

3. Identify \( i \), unknown. \( i = ? \)

4. Identify \( n \). \( n = 12 \)

5. Identify \( A \). \( A = 520 000 \)

6. Substitute. \[ 520 000 = 228 000(1 + i)^{12} \]

7. Divide by the principal (in this case, 228 000). \[ \frac{520 000}{228 000} = (1 + i)^{12} \]

8. Evaluate left side. \[ 2.280 701 754 = (1 + i)^{12} \]

9. Take the appropriate root of the left side to transpose the power on the right side (in this case, the 12th root). \[ \sqrt[12]{2.280 701 754} = 1 + i \]
10. Evaluate left side.
   Use the \( \sqrt[n]{\text{ key on your calculator. (Some calculators may have } \sqrt[n]{\text{ or } x^{1/n}.)} \) Leave 2.280 701 754 on the display from the previous calculation and then press \( \text{ANS} \).

11. Transpose to find \( i \).
12. Express as a decimal correct to one decimal place.

(c) 1. Write the formula.
2. Identify \( P \).
3. Identify \( i \).
4. Identify \( n \), unknown.
5. Identify \( A \).
7. Simplify the right side (in this case, evaluate \( 1 + 0.075 \)).
8. Divide both sides by the principal (in this case, 450).
9. Evaluate the left side.
10. Use a calculator or spreadsheet to try different values for \( n \) until you find a satisfactory answer (in this case, until \( 1.075^n \approx 2 \)).

\[
0.071 122 251 = 1 + i
\]

\[ i = 0.071 122 251 \]

\[ R = 7.1\% \]

(c) \[ A = P(1 + i)^n \]
\[ P = 450 \]
\[ i = 0.075 \]
\[ n = ? \]
\[ A = 900 \]
\[ 900 = 450(1 + 0.075)^n \]
\[ 900 = 450(1.075)^n \]
\[ \frac{900}{450} = (1.075)^n \]
\[ 2 = (1.075)^n \]

\[
\begin{array}{|c|c|}
\hline
n & 1.075^n \\
\hline
8 & 1.783 477 826 \\
9 & 1.917 238 662 \\
10 & 2.061 031 562 \\
\hline
\end{array}
\]

After ten years the stamp will be worth at least $900.

Remember, the compound interest formula is:
\[ A = P(1 + i)^n \]

**exercise 1.2 Compound interest**

**Skills**

1. Calculate the total amount owing on a loan of $7000 after two years, if the 16% interest p.a. is:
   (a) compounded annually
   (b) calculated as simple interest.
2 Calculate the total amount owing on a loan of $6000 after three years, if the 14% interest p.a. is compounded annually.

3 How much will a coin, currently valued at $1300, be worth after two years if it appreciates (increases) in value by 12.5% p.a.?

4 (a) The amount an investment of $8200 amounts to after two years, if 9.5% p.a. interest is compounded annually, is closest to:
A $1558  B $9832  C $7790  D $9758  E $1632
(b) The amount an investment of $4000 amounts to after two years, if 3.5% p.a. interest is compounded annually, is closest to:
A $4280  B $280  C $285  D $4192  E $4285

5 Calculate the total amount owing after two years on a loan of $16 250 if the 11.25% interest p.a. is compounded:
(a) annually  (b) half-yearly.

6 (a) $4245 at 6.5% p.a. compound interest, compounded annually over eight years, will amount to:
A $4245 (1 + 0.065)^8  B $4245 (1 + 0.65)^8
C $4245 (1 + 0.065) \times 8  D $4245 (1 + 0.065)^8 - $4245
E $4245 (1 + 0.065)^8
(b) If a loan of $5800 is made at 16% p.a. compounded every half-year, over six years the debt will grow to:
A $5800 (1 + 0.16)^6  B $5800 (1 + 0.08)^6  C $5800 (1 + 0.08)^12
D $5800 (1 + 0.16)^12  E $5800 (1 + 0.8)^12

7 Nicola’s salary has increased by 7% p.a. over the past four years to $41 500 p.a. Calculate her salary four years ago, to the nearest dollar.

8 Calculate how much would need to be invested at 8% p.a. compounded each half-year to accumulate to $9600 in six years.

9 How much would Li have to deposit in order to receive $10 000 in seven years if she places her money in an account that pays 8.8% interest p.a., compounded quarterly?

10 Sales of $26 900 grow to $78 000 in 11 years. Calculate the percentage growth p.a.

11 After how many years will a $2400 sapphire ring be worth at least $8000 if it increases in value by 10.5% p.a.?
12 After how many years will a $40 000 block of land be worth at least $90 000 if it increases in value by 8.5% p.a.?

13 After how many years will a $3400 porcelain dinner set be worth at least $5000 if it increases in value by 9.7% p.a.?

14 (a) Costs in a business are growing at 8% p.a. Currently they run at $780 per week. Seven years ago they were:

A \[ \frac{780}{1.08^7} \]

B \[ 780 \times 1.08^7 \]

C \[ 780 \times 1.08^7 - 780 \]

D \[ \frac{780}{1.001 \, 538} \]

E \[ 780 \]

(b) A deposit accumulates to $4500 in nine months at 12% p.a. compounded quarterly. The initial deposit, in dollars, was:

A \[ \frac{4500}{1.12^{3/4}} \]

B \[ 4500 \times 1.04^3 \]

C \[ \frac{4500}{1.03^3} \]

D \[ \frac{4500}{1.03^4} \]

E \[ \frac{4500}{1.33} \]

Applications

15 Calculate the total amount owing on a car loan of $16 000 after two years, if the 14% interest p.a. is: 
(a) compounded annually 
(b) compounded every half-year.

16 Find values for the amount of an investment and the annual interest rate, given that the investment compounds annually for two years to give a final value within $100 of $15 000.

17 (a) Find values for \( P \) and \( i \) that would result in an \( A \) value within $10 of $12 000, if \( n = 5 \).

(b) Find values for \( A \) and \( i \) that would result in a \( P \) value within $10 of $10 000, if \( n = 7 \).
(c) Find values for $P$ and $A$ that would result in an $i$ value between 0.065 and 0.07, if $n = 6$.

18 (a) $20 000 is to be invested for three years. Calculate the interest earned on each of the following to find which would give the best return.

(i) 5% p.a. compounded quarterly

(ii) 4.5% p.a. compounded every two months

(iii) 4% p.a. compounded monthly

(b) Does the size of the investment alter which option in part (a) would give the best return?

Analysis

19 $5000 is invested for four years with the interest compounded quarterly. For the first two years the interest rate is 4% p.a. and for the other two years it is 4.8% p.a.

(a) How much will the investment be worth at the end of the four years?
(b) How much interest will be earned?
(c) Would the investor be better off to invest for the whole four-year period at 4.4% p.a.? How much interest would this investment earn?
(d) How much interest would be earned if the first two years were at 4.8% p.a. and the rest at 4% p.a.?
(e) The conditions of investment change and interest is now compounded annually. What would the interest rate need to be to earn the same amount of interest as found in part (b)?
(f) The interest rate is now 5% p.a. compounded annually. After how many years will the investment be at least as great as the result shown in part (a)?

Investigating compound interest and the ‘rule of 72’

The rule of 72 is a rule of thumb used to find the period of time over which an investment doubles in value. The best way to understand the rule is through an example. Tanya purchases a block of land for $100 000. How many years will it take for the land to double in value if its value increases at a rate of 8% p.a.?

Using the rule of 72, we divide 72 by 8 to get 9. That is, it will take nine years for the land to double in value. We can verify this using the compound interest formula:

\[ A = P(1 + i)^n \]

\[ = 100 000(1 + 0.08)^9 \]

\[ = 199 900.46 \]

\[ = $200 000 to the nearest thousand dollars \]
If the rate of increase in value were 12% p.a., it would take only six years for Tanya's block of land to double in value, since 72 ÷ 12 = 6.

1 Verify this result using the compound interest formula.

2 We can graph the rule of 72 on a graphics calculator like this:
Let represent the number of years it takes for an investment to double in value and let represent the percentage rate of interest per annum. The mathematical relationship between and is

Graph this rule on your calculator. Set the calculator’s window so that ranges from 0 to 10 and from 0 to 80. Trace along the curve to inspect values of and . (You may want to alter so that your calculator gives ‘nice’ values.)

3 Use the graph (with the TRACE cursor) to complete the second column of the table below. Use the compound interest formula to complete the third column of the table.

<table>
<thead>
<tr>
<th>Interest rate per annum,</th>
<th>Years for investment to double in value,</th>
<th>Actual value of $100 000 after years using the compound interest formula</th>
<th>Previous column subtract $200 000 (A − $200 000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[A = P(1 + i)^n]</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

Between which two values of do you think the rule of 72 gives a result of $200 000, to the nearest dollar? Zoom in on your graph until you have the required accuracy for your answer.

**Extension**

How accurate is the rule of 72? Investigate the accuracy of the rule of 72 by using the calculator’s list feature (or by creating a spreadsheet) based on the table on the previous page. Let the values start at 1 and end at 50, increasing in steps of 1. Add a fifth column for the percentage error. Between which two values of did the rule of 72 give accurate results? What level of accuracy is acceptable?
DIY summary

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1. The rising cost of goods, and the accompanying rise in wages and salaries, is caused in part by [complement, compound interest, conjectures, continuous, deduction, discrete, effective profit, element, empirical induction, finite set, induction, infinite set, inflation, intersection, Karnaugh map, mathematical induction, union, universal set].

2. When the interest earned on an investment is added to the principal for the next interest period we are dealing with [compound interest, conjectures, continuous, deduction, discrete, effective profit, element, empirical induction, finite set, induction, infinite set, inflation, intersection, Karnaugh map, mathematical induction, union, universal set].

3. The difference in the cost price of an item, adjusted for inflation, and the selling price of the item is called [complement, compound interest, conjectures, continuous, deduction, discrete, effective profit, element, empirical induction, finite set, induction, infinite set, inflation, intersection, Karnaugh map, mathematical induction, union, universal set].

4. If the annual interest rate is 5%, the interest rate for a compounding period of one quarter will be [compound interest, conjectures, continuous, deduction, discrete, effective profit, element, empirical induction, finite set, induction, infinite set, inflation, intersection, Karnaugh map, mathematical induction, union, universal set].

VELS personal learning activity

1. Explain clearly, with examples, how simple interest differs from compound interest.

2. Inflation affects the price of goods.
   (a) If you know the price that an item cost 3 years ago, explain how to work out how much you would expect to pay for it now, accounting for inflation only.
   (b) If you know the cost of an item now, describe how to work out what you would have expected it to have cost 5 years ago.

Skills

1. The following charges apply for long distance calls, over various distances, with a particular company.

<table>
<thead>
<tr>
<th>Call type</th>
<th>Connection fee (cents)</th>
<th>Day rate 8 a.m.–6 p.m. Mon.–Fri.</th>
<th>Night rate 6 p.m.–10 p.m. Mon.–Thu.</th>
<th>Economy rate Other times</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDD2</td>
<td>12.0</td>
<td>0.20465</td>
<td>0.14749</td>
<td>0.09259</td>
</tr>
<tr>
<td>NDD3</td>
<td>12.0</td>
<td>0.43483</td>
<td>0.31339</td>
<td>0.18853</td>
</tr>
<tr>
<td>NDD4</td>
<td>12.0</td>
<td>0.44950</td>
<td>0.37333</td>
<td>0.21729</td>
</tr>
</tbody>
</table>
Find the cost of each of these calls (in dollars).
(a) an NDD2 call for 200 seconds at 7 p.m. Friday
(b) an NDD4 call for 2 1/2 minutes at 9 a.m. Monday
(c) an NDD3 call for 6 minutes at 9 p.m. Thursday
(d) an NDD2 call for 7 1/2 minutes at 3 p.m. Monday
(e) an NDD3 call for 900 seconds at 6.30 p.m. Sunday

2 Nina’s salary has increased by 6% p.a. over the past three years to $40 500 p.a. Calculate her salary three years ago.

3 Calculate the rate of interest p.a. (correct to one decimal place) that would allow $6700 to accumulate to $10 000 in five years if interest is compounded each quarter.

4 Assume an inflation rate of 5.6% p.a. In four years, an ice-cream, currently $1.20, will cost:
A $\frac{1.20}{(1.056)^4}$
B $\frac{1.20}{(1.56)^4}$
C $1.20(1.056)^4$
D $1.20(1.56)^4$
E $1.20(1.014)^{16}$

5 What is the effective percentage profit at time of purchase of an Edwardian dining suite, bought for $2800 and sold for $4600 in 3.5 years’ time? Assume an inflation rate of 4.9% p.a. and a commission paid on the sale price of 10%.

6 On a Venn diagram, show the region represented by each of the following:
(a) $A \cap B \cup C'$
(b) $A' \cap B \cap C'$
(c) $A \cap B' \cap C$

7 Describe an event that would render each of the following conjectures invalid, and an example if you know one:
(a) all birds fly
(b) the square root of a number can always be found
(c) the highest common factor of two numbers is always 2
(d) when odd numbers are added the result is always divisible by 2.

Applications
8 Electricity costs 14.11 cents per kWh for the first 1020 kWh and 14.87 cents per kWh for the remaining use. Find the number of kWh used for:
(a) $110$
(b) $173$
(c) $266$

Applications
9 (a) How much interest will be earned on an investment of $15 000 at 3% p.a. compounded annually for three years?

(b) How much greater would the interest be if it were compounded monthly instead of annually? Express your answer:
(i) in dollars
(ii) as a percentage.

(c) How much greater would the interest be if it were compounded daily instead of annually? Express your answer:
(i) in dollars
(ii) as a percentage.

10 The universal set is the numbers from 1 to 15, Set A is the set of even numbers from 2 to 14 inclusive, and Set B is the set of prime numbers from 1 to 15.

(a) State the elements of each of the sets.

(b) Draw up a Karnaugh map that shows the elements in each of the intersections.

(c) Show that the Karnaugh map contains all the elements in the universal set.

(d) Describe the universal set in terms of the union of the intersection of the sets A and B, and their complements.

Analysis

11 The following table compares costs associated with mobile phone plans available from a number of different companies. Each plan is designed for the high use consumer who would make between 50 and 100 calls per week. Each plan has a 24-month contract, except for Orange Mobile which has no contract. For each plan the billing increment is 30 seconds.

<table>
<thead>
<tr>
<th>Service provider</th>
<th>Plan name</th>
<th>Free calls included</th>
<th>Minimum monthly charge</th>
<th>Flagfall</th>
<th>Fixed line rates</th>
<th>Mobile to mobile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Peak</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Off-peak</td>
<td>Peak 30 sec</td>
<td>Off-peak 30 sec</td>
</tr>
<tr>
<td>B Clear and Simple</td>
<td>B Active 149</td>
<td>$149.00</td>
<td>$149.00</td>
<td>18.7c</td>
<td>18.7c</td>
<td>22c</td>
</tr>
<tr>
<td>New Tel</td>
<td>Hear It 127</td>
<td>$127.00</td>
<td>$127.00</td>
<td>15c</td>
<td>15c</td>
<td>17c</td>
</tr>
<tr>
<td>Optus</td>
<td>Your Call 150</td>
<td>$150.00</td>
<td>$150.00</td>
<td>18.7c</td>
<td>18.7c</td>
<td>18.7c</td>
</tr>
<tr>
<td>Orange Mobile</td>
<td>AnyTime 100</td>
<td>$100.00</td>
<td>$100.00</td>
<td>22c</td>
<td>22c</td>
<td>30c for first 10 min, then 18c/30 sec (no flagfall). 18c/30 sec to other networks</td>
</tr>
</tbody>
</table>

(a) What do you notice about the Peak and Off-peak rates for each of these plans?
(b) Find the cost of the following Peak calls for each network. (For Orange Mobile, assume that the call is to another network.)

(i) 20 s  
(ii) 55 s  
(iii) 130 s  
(iv) 250 s

(c) On the basis of the information presented here, which of the first three plans seems to be the best?

d) How does Orange Mobile complicate the choice of plan?

e) What other features might need to be considered when making your choice?

12 The following table shows the average annual inflation rate in Australia for each decade from the 1950s to the 1990s.

<table>
<thead>
<tr>
<th>Decade</th>
<th>1950s</th>
<th>1960s</th>
<th>1970s</th>
<th>1980s</th>
<th>1990s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual inflation rate (%)</td>
<td>6</td>
<td>2</td>
<td>10.5</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Find the cost at the end of each decade of an item purchased at the beginning of the decade for $5000 if the price increase matched inflation.

(b) Was the increase in price in the 1960s one-third of the increase in the 1950s? What do you conclude from this?

(c) Find the cost at the end of the 1990s of an item bought at the beginning of the 1950s for $1000 if its price increase matched inflation.

(d) Find the actual percentage increase for the item in part (c).

(e) Repeat parts (c) and (d) for items costing:

(i) $2000  
(ii) $5000  
(iii) $10 000.

(f) What do you conclude from this?

1 Find the simple interest earned, correct to the nearest cent, on each of these investments.

(a) $250 at 3% p.a. for 5 years  
(b) $590 at 5.2% p.a. for 3 years  
(c) $830 at 2.6% for 18 months

2 Find the total surface area, correct to two decimal places where necessary, of:

(a) a cube with side length 5 cm  
(b) a cylinder with a diameter of 14 cm and a length of 20 cm.
3 Find the missing side lengths, correct to one decimal place where necessary.
   (a) \[ \text{3 cm} \quad \text{5 cm} \quad x \text{ cm} \]
   (b) \[ \text{7 cm} \quad \text{11 cm} \quad y \text{ cm} \]

4 Expand and simplify the following expressions.
   (a) \((3x + y)(2x - 6y)\)
   (b) \((a - 3b)(b - 3a)\)

5 Find the value of the following, correct to four decimal places.
   (a) \(\tan 26^\circ\)
   (b) \(\cos 17^\circ\)
   (c) \(\sin 59^\circ\)

6 Solve the following equations.
   (a) \(3x + 7 = 10\)
   (b) \(4x - 5 = -15\)
   (c) \(4x + 5 = 3x + 6\)

7 Draw a box-and-whisker plot for the following data, which represents the number of flaws in each item during a production run.
   
<table>
<thead>
<tr>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

8 Find the marked angle in each triangle.
   (a) \[ \text{50°} \]
   (b) \[ \text{40°} \]
   (c) \[ \text{105°} \]

9 Solve the following equations.
   (a) \(x^2 + 8x + 15 = 0\)
   (b) \(x^2 - 3x + 2 = 0\)

10 A card is drawn at random from a normal pack of 52 playing cards. Find the probability that the first card drawn is:
   (a) black
   (b) a spade
   (c) a red 3
   (d) a 5
   (e) a 4 or an 8
   (f) the King of diamonds.

11 Convert the following decimals to fractions.
   (a) \(0.314\)
   (b) \(1.22\)
   (c) \(3.046\)

12 What is the radius of the circle that has a circumference of 12.3 cm? Give your answer correct to two decimal places.
DIY summary

Copy and complete the following using the words and phrases from the list where appropriate to write a summary for this chapter. A word or phrase may be used more than once.

1. The number 2 is a ______ of 12a and 8, while the ______ of 12a and 8 is 4.
2. $ax^2 + bx + c$ is called a ______. If the value of $c$ is zero, $ax^2 + bx$ is still a ______.
3. To obtain the answer $6r + 12$ from $6(r + 2)$, we use the ______.
4. The first thing to do to factorise $50x^2 - 6y^2$ is to take out a ______ of 2. We can then use the ______ method to factorise the expression.
5. When $x^2 - 4x - 6$ is written as $(x - 2 - \sqrt{10})(x - 2 + \sqrt{10})$, the method used is called ______.
6. When we ______ the expression $2ax + 2ay + 3x + 3y$, it becomes $(2a + 3)(x + y)$.
7. $a$ and $3b$ are ______ of $4a$ and $6bc$ respectively.
8. $(3r - 2)^2$ is an example of a ______.
9. In ______, $3w + 9$ becomes $3(w + 3)$.
10. The numerator of the ______ ______ $\frac{4x^2 + 2}{3x^2 + 5x + 1}$ is a ______ and the denominator is a ______.

VELS personal learning activity

You have learned to factorise using common factors, the difference of two squares, grouping, quadratic trinomials and completing the square.

1. Write clear instructions completely in your own words about how to factorise using each of these types of factorisations.
2. Explain how you would know which to use.
3. Make up a factorised expression; e.g. $2(x + 1)^2(x - 3)$ and expand it.
4. Swap expanded expressions with a partner for them to factorise. When complete, discuss the answers together. Remember, there may be a problem with the expansion.
Liquid loss

Investigating and designing

A new dry-cleaning machine has been designed. At the end of each cleaning cycle, the dry-cleaning liquid will be reduced by evaporation and condensation. Each time the machine is used there is a loss of 2% of the dry-cleaning liquid.

1. Initially, the machine is filled with 1000 mL of dry-cleaning liquid. How much liquid will remain after the machine has been used once, twice and five times?

2. Develop a formula that uses indices to determine the amount of liquid left after \( n \) uses.

Producing

3. Use your formula to check your answers from 1.

4. Show that after 20 uses the quantity of liquid remaining is approximately two-thirds of the original quantity.

5. How much liquid will remain after 40 uses? What proportion is this of the original quantity?

6. If the original quantity of liquid was 2000 litres, change your formula appropriately and redo the calculations for 5.

Analysing and evaluating

7. Make a comparison of your answers to 5 and 6. What do you notice?

8. When the quantity of liquid is reduced to \( \frac{1}{4} \) of the original amount it is time to replace the liquid. After how many uses will the liquid need to be replaced?

9. What amount of liquid would initially be required if the amount remaining when the liquid needs replacing is 200 mL?