1.5 Primes, powers and square roots

Prime numbers

We give special names to numbers depending on how many factors they have.

A **prime number** has exactly two factors: itself and 1.
A **composite number** has more than two factors.
1 is a special number—neither prime nor composite.

Prime factors and factor trees

A **prime factor** of a number is a factor that is also prime. Every composite number can be expressed as a product of prime numbers.

A **factor tree** is a good way to find these prime factors.

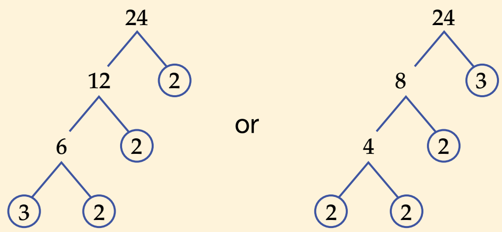
worked example 10

Write 24 as a product of prime factors.

Steps

1. Break the number into a pair of factors (not using 1), then break each subsequent factor into a pair of factors. Circle any prime factors as they appear, and stop the branch at this point. (Two possible trees are shown.)
2. Write the number as a product of the circled factors.

Solution



$24 = 2 \times 2 \times 2 \times 3$

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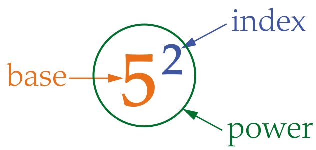
Powers

Powers are a short way of writing repeated factors. 5^2 is an example of a **power**. The small 2 is the **index** and tells us how many factors of 5 there are. The 5 is called the **base**.



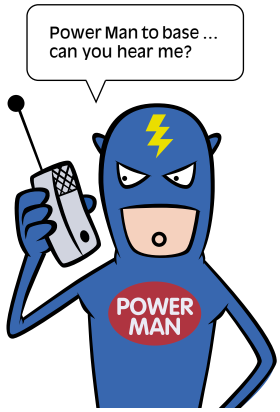
For example:

$$4^2 = 4 \times 4$$
$$5^3 = 5 \times 5 \times 5$$
$$7^4 = 7 \times 7 \times 7 \times 7$$
$$9^5 = 9 \times 9 \times 9 \times 9 \times 9$$

For 4^2 we say 'four squared' or 'four to the power of two' or 'four to the second power'.

For 5^3 we say 'five cubed' or 'five to the power of three' or 'five to the third power'.

For 7^4 we say 'seven to the power of four' or just 'seven to the fourth'.



Another word for index is **exponent**. Numbers written using index notation are in index form or exponential form.

4^3 is in index form.

$4 \times 4 \times 4$ is in **expanded** form.

worked example 11

(a) Write out 4^3 in expanded form and work out the answer.

(b) Write out 7^4 in expanded form and work out the answer.

Steps	Solutions
(a) 1. Write 4^3 in expanded form.	(a) $4^3 = 4 \times 4 \times 4$
2. Write down the answer.	$= 64$
(b) 1. Write 7^4 in expanded form.	(b) $7^4 = 7 \times 7 \times 7 \times 7$
2. Write down the answer.	$= 2401$

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Powers on calculators

Many calculators have a key to help you work quickly with powers, usually [x^y] or [^]. For example, if you want to work out 12^3, press [1] [2] [x^y] [3] [=]. To work out 5^2, simply press [5] [x^2].

Squares and square roots

A number is called a **perfect square** if it can be found by multiplying another number by itself (i.e. by **squaring** the other number).

Finding the **square root** is the opposite to squaring a number. For example, to find the square root of 16, you have to think of a number that when multiplied by itself gives you 16. The number is 4 since $4 \times 4 = 16$.

This means the square root of 16 is 4. We write this as $\sqrt{16} = 4$.

You can use your calculator to help with some calculations. Your calculator will have a key like this: [$\sqrt{\ }$].

To find $\sqrt{2304}$, press [$\sqrt{\ }$] [2] [3] [0] [4] [=] or [2] [3] [0] [4] [$\sqrt{\ }$] depending on the type of calculator. Then read off the answer: 48.

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exercise 1.5 Primes, powers and square roots

Preparation: Exercise 1.4

1 Complete these factor trees, then write each number as a product of its prime factors.

Worksheet C1.8

(a) 16 → 8, ___

$16 = \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_$

(b) 50 → 2, ___

$50 = \_\_\_ \times \_\_\_ \times \_\_\_$

(c) 100 → 10, ___

$100 = \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_$

(d) 96 → 3, ___

$96 = \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_$

2 Express each number as a product of its prime factors.

Interactive
Hint
eQuestions
eQuestions
Worksheet C1.9

(a) 8 **(b)** 12 **(c)** 18 **(d)** 26
(e) 20 **(f)** 48 **(g)** 68 **(h)** 44
(i) 64 **(j)** 72 **(k)** 108 **(l)** 144
(m) 140 **(n)** 1000 **(o)** 800 **(p)** 400

3 Find three different numbers that can each be expressed as a product of four prime factors. (The product may contain repeated factors.)

4 Simplify these with your calculator, using the shortest method possible.

Hint

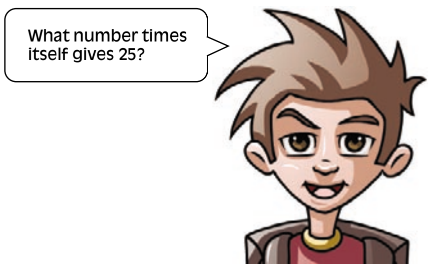
(a) 21^2 **(b)** 18^2 **(c)** 48^2 **(d)** 61^2
(e) 75^2 **(f)** 88^2 **(g)** 540^2 **(h)** 439^2
(i) 2305^2 **(j)** 1766^2 **(k)** $43^2 - 203$ **(l)** $74^2 - 1341$
(m) $68^2 \times 14^2$ **(n)** $37^2 \times 22^2$ **(o)** $26^2 + 41^2$ **(p)** $23^2 - 14^2$

5 What number do you have to square to get each of these?

Hint
eQuestions

(a) 9 **(b)** 4 **(c)** 49 **(d)** 64
(e) 81 **(f)** 25 **(g)** 144 **(h)** 10 000

6 Evaluate the following.



(a) $\sqrt{25}$ **(b)** $\sqrt{49}$
(c) $\sqrt{36}$ **(d)** $\sqrt{64}$
(e) $\sqrt{100}$ **(f)** $\sqrt{121}$
(g) $\sqrt{196}$ **(h)** $\sqrt{169}$
(i) $\sqrt{1}$ **(j)** $\sqrt{0}$
(k) $\sqrt{4900}$ **(l)** $\sqrt{400}$
(m) $\sqrt{1600}$ **(n)** $\sqrt{2500}$
(o) $\sqrt{360\ 000}$ **(p)** $\sqrt{1\ 000\ 000}$

eQuestions

7 Write each of these in index form. (Don't work out the answer.)

Hint

(a) $8 \times 8 \times 8$ **(b)** $4 \times 4 \times 4 \times 4 \times 4 \times 4$
(c) $9 \times 9 \times 9 \times 9$ **(d)** $7 \times 7 \times 7$

(e) $5 \times 5 \times 5 \times 5 \times 5 \times 5$
(f) $9 \times 9 \times 9 \times 9 \times 9$
(g) $12 \times 12 \times 12 \times 12 \times 12$
(h) $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
(i) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
(j) $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$
(k) seventeen cubed **(l)** thirteen to the power of seven
(m) eight to the fourth power **(n)** nine to the sixth
(o) eleven to the seventh **(p)** nine to the third

eQuestions

8 Write each of these out in expanded form, then work out the answer.

Hint

(a) 2^3 **(b)** 2^4 **(c)** 2^6 **(d)** 2^5
(e) 1^8 **(f)** 0^7 **(g)** 0^6 **(h)** 1^7
(i) 10^3 **(j)** 10^5 **(k)** 6^4 **(l)** 8^4
(m) 11^3 **(n)** 12^4 **(o)** 4^6 **(p)** 6^6

9 What number do you have to cube to get each of these?

Hint

eQuestions

(a) 1 **(b)** 27 **(c)** 8 **(d)** 64
(e) 8000 **(f)** 1000 **(g)** 125 **(h)** 1 000 000

10 $\sqrt[3]{\ }$ means *cube root*. The cube root of 27 is 3 because $3 \times 3 \times 3 = 27$.
Evaluate, without using a calculator.

Hint

(a) $\sqrt[3]{8}$ **(b)** $\sqrt[3]{64}$ **(c)** $\sqrt[3]{1000}$ **(d)** $\sqrt[3]{1}$
(e) $\sqrt[3]{0}$ **(f)** $\sqrt[3]{125}$ **(g)** $\sqrt[3]{8000}$ **(h)** $\sqrt[3]{27\,000}$

11 **(a)** Arrange these numbers in ascending order (smallest to largest).

Hint

4^5, 5^4, 1^{200}, 10^3, 4^6, 5^5

(b) Arrange these numbers in descending order (largest to smallest).
100^2, 10^5, 1^{1000}, 0^{100}, 3^2, 2^3

12 **(a)** Copy and complete:

$$10^1 = 10$$
$$10^2 = 10 \times 10 = 100$$
$$10^3 = 10 \times 10 \times 10 = 1000$$
$$10^4 =$$
$$10^5 =$$

(b) The number 10^{100} was called a **googol** by the mathematician Edward Kasner.

(i) How many times is 10 multiplied by itself to get a googol?

(ii) Look at the pattern in part **(a)**. If you were to work out what a googol is equal to, how many zeros would follow the 1?

(c) If you raise the number 10 to the power of a googol, you get a number called a **googolplex**.

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(i) How many times is 10 multiplied by itself to get a googolplex?

(ii) If you were to work out what a googolplex is equal to, how many zeros would follow the 1? How much time do you think you save by writing a googolplex in index form?

13 Without using your calculator, find between which two consecutive whole numbers each of the following lies.

(a) $\sqrt{10}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{20}$ **(d)** $\sqrt{62}$

(e) $\sqrt{99}$ **(f)** $\sqrt{2}$ **(g)** $\sqrt{70}$ **(h)** $\sqrt{108}$

Hint

14 Without using your calculator, find between which two consecutive whole numbers each of the following lies.

(a) $\sqrt[3]{10}$ **(b)** $\sqrt[3]{52}$ **(c)** $\sqrt[3]{1001}$ **(d)** $\sqrt[3]{30}$

(e) $\sqrt[3]{5}$ **(f)** $\sqrt[3]{71}$ **(g)** $\sqrt[3]{120}$ **(h)** $\sqrt[3]{2}$

Hint

15 **(a)** Use your calculator to answer TRUE or FALSE to each statement.

(i) 4^6 is bigger than 6^4. **(ii)** 2^{10} is bigger than 10^2.

(iii) 3^9 is bigger than 9^3. **(iv)** 19^2 is bigger than 2^{19}.

Hint

(b) Look at your answers for part **(a)** and *without* using your calculator answer TRUE or FALSE to each of these statements.

(i) 9^8 is bigger than 8^9. **(ii)** 2^{100} is bigger than 100^2.

16 Use your calculator to find the following through trial and error.

(a) A number that when multiplied by itself three times gives 343

(b) A number that when multiplied by itself three times gives 1728

(c) A number that when multiplied by itself three times gives 2744

(d) A number that when multiplied by itself three times gives 39 304

(e) A number that when multiplied by itself four times gives 4096

(f) A number that when multiplied by itself five times gives 161 051

Hint

Worksheet C1.10

17 **(a)** Copy the following pattern and complete it using your calculator.

$11^2 =$ ____ $111^2 =$ ____ $1111^2 =$ ____

Hint

(b) Look at the pattern in part **(a)** and *without* using a calculator copy and complete the following.

$11\,111^2 =$ ____ $111\,111^2 =$ ____ $1\,111\,111^2 =$ ____

Homework 1.2

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Do these in your head as quickly as you can and write down the answers.

Time target: 2 minutes

1 17×6

2 $112 - 38$

3 $1\frac{4}{5} - \frac{3}{5}$

4 $0.2 + 1.4$

5 $3600 \div 900$

6 \$2.60 + \$9.50

7 What time is it 5 hours and 40 minutes after 9.17 a.m.?

8 $\frac{1}{2}$ of \$27.00

9 How many odd numbers are there between 20 and 36?

10 Find the missing number: 816, 408, ____ , 102.

maths in action

Prime numbers versus terrorism

Terrorism has cost many innocent lives so far this century, and mathematics has a role to play in stopping terrorist acts before they happen. Terrorists are able to commit these acts because they can plan and communicate in secret. Government intelligence agencies all over the world try to intercept terrorist computer e-mails and decode them.

At the time of the terrorist attacks in 2001, *encryption* or coding techniques existed that couldn't be cracked. These encryption techniques are based on prime numbers.

All encryption is based on a *key*, which is a word or a number. These keys give the information needed to decode or decrypt the message. The keys currently most commonly used to encrypt e-mails are numbers that are the products of two prime numbers. To crack the code, you have to work out the original two prime numbers. When the numbers involved are very large, this is almost impossible to do because there are so many possibilities. The largest prime ever found has over 2 million digits in it, and there are always new primes being found. Even the most powerful computers in the world would take centuries to find the two primes involved in encryption. Multiplying two primes is an effective key for encryption because it is easy to do in one direction (multiplication), but extremely difficult to do in the other direction (finding factors).

Governments are hoping that in the future quantum computers that can process calculations simultaneously will be able to find the two prime numbers quickly and enable them to crack the terrorists' codes. Breaking an encryption code is called *decryption*.

Questions

1 Which two prime numbers were multiplied together to give each of these encryption keys?

(a) 77	**(b)** 38	**(c)** 65
(d) 202	**(e)** 143	**(f)** 2183

2 Multiply the following prime numbers together to get encryption keys.

(a) 3 and 17	**(b)** 7 and 31	**(c)** 47 and 73
(d) 101 and 103	**(e)** 131 and 727	**(f)** 313 and 93 139

3 **(a)** Write out the first five prime numbers.
(b) Write out the six smallest keys that you can create by multiplying two primes.
(c) How many factors does each key have?
(d) Do you think every key created by multiplying two primes together will have this number of factors? Why?

4 Here's a simplified version of how decryption using prime numbers works. Suppose the encrypted message is W L R G Y S B E M R K C R E B J C V and you know the key is 713. You have to find the two prime numbers that multiply together to give 713. This is reasonably easy for such a small number. The answer is 23 and 31.
These digits tell you how much the letters in the original message have been shifted backwards along the alphabet to give the coded message. You match these digits to the code as follows:

W	*L*	*R*	*G*	*Y*	*S*	*B*	*E*	*M*	*R*	*K*	*C*	*R*	*E*	*B*	*J*	*C*	*V*
2	3	3	1	2	3	3	1	2	3	3	1	2	3	3	1	2	3

The way decrypting this code works is that the number tells you how many letters to shift each letter forwards along the alphabet. The 2 under the W means shift two letters forward through the alphabet from W, so the uncoded message, usually called the *plaintext*, has Y as its first letter. For the second letter you find the letter three places on from L in the alphabet, which gives O as the second letter of the plaintext.
(a) Continue decrypting the code to get the entire original plaintext message.
(b) Describe what you did with Y 2.
(c) Why couldn't you use any number (e.g. 120) as a key?

5 Decode this message given the key 943.
K X P G Q F O N D M N H K B E L N L N S Y K Z D
Note that you need to work out which order to put the two primes in. Only one will give you the message.

6 Encrypt your own message using the prime number system. Use only prime numbers under 100 and keep the message under 25 letters. Give your coded message and key to someone to decrypt.

Research

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Find out about the GIMPS (Great Internet Mersenne Prime Search) project and present a report on how prime numbers are used in e-mail encryption. Discuss whether governments should ban encryption and make it possible for intelligence agencies to read all e-mails.

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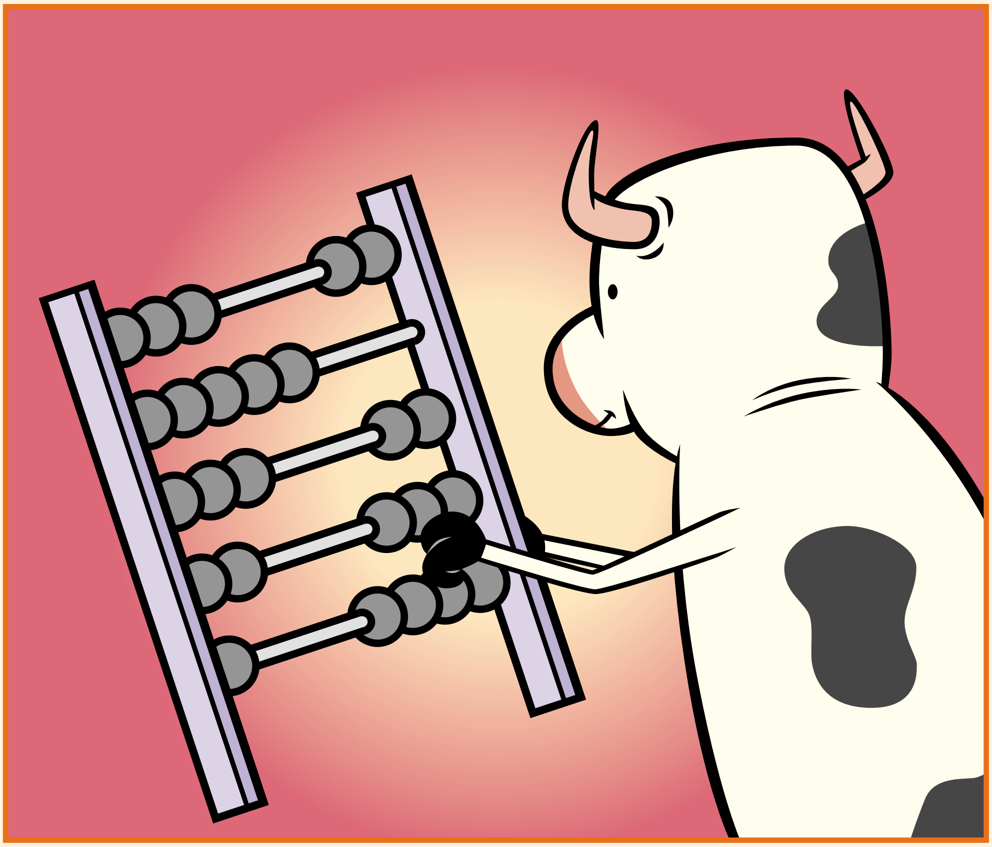

Simplify the following with the help of a calculator, and break the code to find the caption.

R $1^4 + 2^2$

C $10^2 + 3^2 - 4^3$

O $(8 + 5^3) \times 16$

A $9^2 \times 10^4$

L $16^3 - 4096$

U $2^{12} + 2^{18}$

W $3^4 - 2^4$

C $2^3 \times 2^2$

T 4×10^3

O $2 \times 10^4 + 7 \times 10^3 + 8 \times 10^2 + 6 \times 10 + 9$

A $36^3 \times 53$

810 000	32	2128	65	–

45	266 240	0	2 472 768	4000	27 869	5

chapter 1 REVIEW

Summary

- Set out solutions to worded problems by putting headings and writing calculations in logical sequence.
- Estimate products and quotients by rounding numbers to the leading digit.
- Order of operations requires bracketed calculations to be done first, working from inner brackets, then multiplication and division as they appear, then addition and subtraction as they appear.
- Primes are numbers with exactly 2 factors: themselves and 1.
- Index notation is a way of writing repeated factors as a power of the factor.
- Taking a square root is the opposite of squaring a number.

FAQs

What is the largest multiple of 9?

Numbers don't have 'largest multiples'. The multiples of 9 continue forever: 9, 18, 27, 36, …

Is 1 a prime number?

No. A prime number has two factors. 1 is neither composite nor prime.

Skills

1 Use rounding to the first digit to estimate these products. 1.2
(a) 3741 × 22 **(b)** 265 × 341 **(c)** 986 × 35

2 Use rounding to the first digit to estimate these quotients. 1.2
(a) 25 736 ÷ 49 **(b)** 96 001 ÷ 17 **(c)** 25 000 ÷ 621

3 Use rounding to the first digit to estimate these, and then use a calculator to work out how far off your estimate was from the exact answer. 1.1, 1.2
(a) 73 – 29 + 5628 **(b)** 17 × 35 × 241 **(c)** 28 × 89 – 2455

4 Work these out using your calculator. 1.3
(a) 258 × (231 – 162) **(b)** [(832 – 94) ÷ 41] × 112

5 Find, without using a calculator: 1.3
(a) 9 ÷ (2 + 1) – 2 **(b)** 12 – 6 × 2 + 11
(c) (13 – 5 × 2) + (20 ÷ 10) **(d)** [5 × (9 + 1)] – 3

6 Choose the correct answer. 1.3
In the calculation of 2 × [30 ÷ (4 – 1)] + 6 the first operation to do is:
A + **B** – **C** × **D** ÷ **E** any of these symbols

7 Choose the correct answer. (5 + 6) × 2 + (15 – 3 × 2) – 6 is equal to: 1.3
A 40 **B** 20 **C** 32 **D** 25 **E** 28

8 Find the first three multiples of: 1.4

(a) 7 **(b)** 10 **(c)** 12 **(d)** 52

9 Find the LCM of: 1.4

(a) 3 and 8 **(b)** 4 and 16 **(c)** 6 and 9 **(d)** 20 and 25

10 Choose the correct answer. The LCM of 2, 5 and 9 is: 1.4

A 10 **B** 18 **C** 45 **D** 90 **E** 15

11 Find all the factors of:

(a) 35 **(b)** 31 **(c)** 44 **(d)** 48 **(e)** 51 **(f)** 100

12 Find the HCF for each of these. 1.4

(a) 11 and 55 **(b)** 90 and 60 **(c)** 72 and 64

13 Choose the correct answer. The HCF of 14, 28 and 56 is: 1.4

A 14 **B** 1 **C** 7 **D** 56 **E** 28

14 Write factor trees and express each number as a product of prime factors. 1.4

(a) 24 **(b)** 30 **(c)** 88 **(d)** 200

15 Write each of these numbers in index form. 1.5

(a) $7 \times 7 \times 7 \times 7 \times 7$ **(b)** ten cubed

(c) five squared **(d)** 12 to the power of 8

16 Write each of these out in expanded form, then work out the answer. 1.5

(a) 5^3 **(b)** 8^4 **(c)** $(3^2 - 2^3) \times 16^2$

17 Evaluate the following. 1.5

(a) $\sqrt{64}$ **(b)** $\sqrt{900}$ **(c)** $\sqrt{225}$

Applications

18 Put brackets into these statements, where necessary, to make them true. 1.3

(a) $4 \times 2 + 3 \div 5 - 1 = 3$ **(b)** $5 + 1 \div 6 + 4 + 2 = 7$

19 **(a)** Copy the following pattern and complete it using your calculator. 1.4, 1.5

$66^2 =$ ____ $666^2 =$ ____ $6666^2 =$ ____

(b) Look at the pattern in part **(a)** and *without* using a calculator copy and complete the following.

$66\,666^2 =$ ____ $666\,666^2 =$ ____ $6\,666\,666^2 =$ ____

20 **(a)** Look at this pattern: $2^2 = 1^2 + 3$. Now copy and complete these. 1.4, 1.5

(i) $3^2 = 2^2 +$ ____ **(ii)** $4^2 = 3^2 +$ ____ **(iii)** $5^2 = 4^2 +$ ____

(b) Describe the relationship between a square number and the square number before it.

(c) Using the relationship established in part **(b)**, complete these.

(i) $12^2 = 11^2 +$ ____ **(ii)** $20^2 = 19^2 +$ ____

21 Is 2^{10} larger or smaller than 1000? Write the difference. 1.5

22 Put these numbers in ascending order: 2^4, $\sqrt{121}$, 10^2, 3^3, $4 \times \sqrt{81}$ 1.5

23 Write each number as a product of its prime factors, in index form. 1.5

(a) 81 **(b)** 128 **(c)** 144 **(d)** 8000

24 Simplify these with your calculator, using the shortest method possible. 1.5

(a) 6^5 **(b)** 13^6 **(c)** 15^3 **(d)** 25^3

(e) 32^3 **(f)** 2^7 **(g)** 3^6 **(h)** 4^5

(i) $16^3 - 4096$ **(j)** $21^4 - 4481$ **(k)** $15^4 - 5625$ **(l)** $36^3 \times 53$

(m) $14^4 \times 14$ **(n)** $19^5 \times 21$ **(o)** $2^{12} + 2^{18}$ **(p)** $3^{10} + 12^4$

REPLAY

1 Set out these calculations in your normal way and work out the answers. Worksheet R1.7

(a) 138 – 97 **(b)** 1902 – 845 **(c)** 5485 – 1099

2 List the numbers if you count by nines, starting at 30 and ending at 75. Worksheet R1.8

3 Copy and complete the following by writing < or > between the numbers. Worksheet R1.9

(a) 1001 ___ 982 **(b)** 3.9 ___ 3.38 **(c)** 0.03 ___ 0.19

4 Copy and complete each of the following by finding the pattern. Worksheet R1.10

(a) 9, 15, 22, 30, ___, ___, ___ **(b)** 1, 3, 7, 15, ___, ___, ___

5 Calculate: Worksheet R1.11

(a) 8000 ÷ 2000 **(b)** 1200 ÷ 4 **(c)** 45 000 ÷ 90

6 Find the missing number that makes each of the following true. Worksheet R1.12

(a) ___ + 7 = 24 **(b)** 4 × ___ = 15 – 3 **(c)** 7 × 0 = 15 – ___

7 Perform the following divisions. Worksheet R1.13

(a) 768 ÷ 3 **(b)** 1404 ÷ 9 **(c)** 7865 ÷ 5

8 List all numbers that 8 goes into that are greater than 70 and less than 150. Worksheet R1.14

9 Simplify: Worksheet R1.15

(a) 2 + 5 × 9 **(b)** 18 ÷ 6 – 3 **(c)** 8 × (15 – 5)

10 Which whole number is each of the following decimals closest to? Worksheet R1.16

(a) 1.9 **(b)** 5.089 **(c)** 37.6001

11 Calculate: Worksheet R1.17

(a) $\frac{7}{9} - \frac{2}{9}$ **(b)** $\frac{3}{4} + \frac{1}{4}$ **(c)** $1\frac{5}{6} - 1$

12 If Celen purchases three tops at $15.50 each, how much change will she receive from a $50 note? Worksheet R1.18

Worksheet C1.11

Worksheet C1.12

Assignment 1